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Activity 1

1 Synopsis.

What we are set about to do. In a previous seminar we saw a simple example on how to estimate things by simulation, using the so-called Monte Carlo method. In this activity you will put to good use your new skill to obtain an approximation of an integral.

What you need. You need to be fully acquainted with the content of the Seminar 1 handout. You will also need access to a computer equipped with R.

2 Problem

Consider the function,

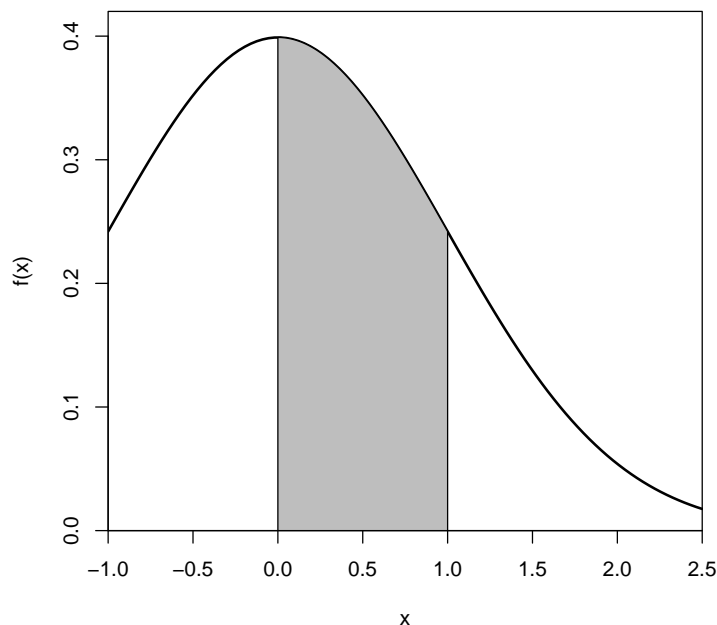
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \quad (1)$$

Yes, in case this looks vaguely familiar to you, this is the $N(0, 1)$ density function. You are required to obtain the approximate integral of such function over the interval $[0, 1]$ (see Figure 1), using simulation.

3 Hints and comments

1. The normal density function has no integral in closed form; you will not be able to solve that integral by parts or any other of the conventional methods. This is an example where simulation offers an approximate solution to a problem which is hard otherwise¹.
2. In order to check your results, you may notice that such area is the probability of the interval $[0, 1]$ in a $N(0, 1)$ distribution, which you can get from the $N(0, 1)$ probability tables as $\Phi(1) - \Phi(0) = 0.841345 - 0.50 = 0.341345$; what you get should be close to that value.

¹The fact that no closed form for the integral exists does not imply that simulation is the only way to compute its value; you might for instance use a Taylor expansion of the integrand and integrate term by term. There are other more efficient functional approximations used in practice. See for instance [1], Sec. 26.2.

Figure 1: Normal density; shaded area is the probability of the interval $[0, 1]$.

3. You may notice that the shaded area of interest is included in a rectangle whose base goes from 0 to 1 and whose height is 0.4 (the maximum of $f(x)$ is easily seen to occur when $x = 0$, at which point $f(x) = 1/\sqrt{2\pi} = 0.39894$). Therefore, you could generate uniform random numbers in the rectangle $[0, 1] \times [0, 0.4]$ and count the proportion of hits in the shaded area, along the lines of what we did in Seminar 1.
4. You can code function $f(x)$ in R quite easily:

```
> x <- 0.7
> (1 / sqrt(2*pi)) * exp(-x^2/2)
```

```
[1] 0.3122539
```

Even that is not necessary, as R provides a function, `dnorm`, which does the same. Compare the value above with:

```
> dnorm(x)
```

```
[1] 0.3122539
```

References

- [1] M. Abramovitz and I. Stegun, editors. *Handbook of Mathematical Functions*. Dover Pub., 1965.