Lecture February, 18, 2013

Case studies to illustrate use of some common distributions

1. Company A is in the business of auto repairs, offering express services. The "star" service is replacement of four tires, which the company claims it is done in time averaging to 65 minutes. When company A applies for a quality assurance certificate, you are selected to perform a quality audit and, among other things, verify their 65-minute mean service time claim.

   You know from experience (and company A is willing to confirm) that service time is exponentially distributed. Unable to check all times for all customers, you collect at random one hundred service times, and find that the total adds up to 6832 minutes. Further, the largest happens to be 312 minutes. With this evidence on hand, you must decide whether the claimed 65-minute mean service time is plausible or not.

   (a) If service time is indeed exponentially distributed, $\exp(\lambda)$, with $m = 65$ minutes, what is λ?

   (b) With said λ, how likely is a service time as large, or larger, as 312 minutes?

   (c) Always with λ as in (a), what would be the probability of no times of 312 minutes, or above, among the one hundred times examined? Do yo think now a 312-minute service time invalidates the company's claim?

   (d) Always with λ as in (a), what is the distribution of the total service time for 100 customers?

   (e) Is the observed total time of 6832 minutes in good agreement with the theoretical distribution in (1d)?

   (f) What would be your decision? Would you accept the company claim, or reject it as implausible given the available evidence?

2. Coastal Insurance Company underwrites insurance for beachfront properties along the Virginia, Carolina and Georgia coasts. It uses the estimate that the probability of a Category III hurricane (sustained winds of more than 110 mph or higher) striking a location of the coast in any one year is 0.05. We will assume the probability of more than one is negligible.

(a) What is the probability that a homeowner purchasing a property with a 30-year mortgage will experience at least one hurricane of Category III during the mortgage period? Use both the binomial and Poisson approximation approach. Would a normal approximation be adequate here?

(b) If Coastal Insurance Company contracts insurance for 1,000 homeowners, in a fairly small area, what is the expected number of claims per year caused by hurricanes of Category III?

(c) What is the *distribution* of the number of claims caused by Category III hurricanes? (Remember all homeowners are "in a fairly small area".)

(adapted from Lind et al. (2011))

Things to investigate on your own.

1. Case 1 makes some assumptions (and raises some questions) that deserve further thought.

   (a) Service times are assumed to be exponentially distributed. This is a rather bizarre assumption.

   (b) Two possible ways of deciding about the company's claim are sketched: one looks only at the largest service time, other at the sum of service times. In both cases, we attempt to assess whether they are plausible given the company's claim.

   This raises some questions: which of the two methods is better? Is there something still better than either? In both cases we are looking at a summary of the sample: the largest and the sum of the service times. Would looking at each service time individually provide some more information?

   You will see systematic treatment of these questions in a few weeks, as we deal with so-called *sufficient statistics*. For now, squeeze your intuition, see what it tells you.

References

D. A. Lind, W. G. Marchal, and S. A. Wathen. *Statistical Techniques in Business and Economics*. McGraw Hill Higher Education, 2011.