Lecture March, 15, 2013.

1. Consider the so-called Rayleigh distribution, whose density is given by:

$$f_X(x;\theta) = \left(\frac{x}{\theta}\right)\exp(-x^2/2\theta) \qquad (0 \leq x < \infty)$$

   Find the maximum likelihood estimator of θ based in a sample on n independent observations. Optional: What would be the moment estimator? (Hint: You need to know the mean of the distribution, or else to work a not-too-difficult integral by parts, see for instance Garín and Tusell (1991) Prob. 10.11.)

2. Consider the Cauchy distribution (Student's t with 1 degree of freedom) shifted by θ. The density is given by:

$$f_X(x;\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2} \qquad (-\infty < x < \infty)$$

   Find out what would be the maximum likelihood estimator of θ based on a sample of size 2.

   (a) What happens when n grows large? (Hint: Here you have an example of an untractable MLE).

   (b) Would a moment estimator of θ be a solution?

   (c) Suggest a possible estimator of θ.

3. Let X be now distributed as a shifted exponential: the density is exponential, but starting at an unknown value η, which has to be estimated along with λ:

$$f_X(x;\lambda,\eta) = \lambda e^{-\lambda(x-\eta)} \qquad (\eta \leq x < \infty)$$

   Find the maximum likelihood estimators of λ and η. (Hint: Beware! Notice that here the support is dependent on one of the parameters. We are back in the situation we faced with the $U(0,\theta)$.)

4. Assume the life of a certain electronic component is exponentially distributed, with unknown parameter λ. From a batch of $n = 200$ units we only know that 100 failed in the first year, 50 the second year and the remaining 50 were still in operation at the beginning of the third year. What is the estimate of λ based in this grouped data?

References

A. Garín and F. Tusell. *Problemas de Probabilidad e Inferencia Estadística.* Ed. Tébar-Flores, Madrid, 1991. In the reserved collection, signature AL-519.2(076) an in the general collection, signature 519.2(076.1) TUS.

S. M. Ross. *Introductory Statistics, Third Edition.* Academic Press, 2010.

L. Ruíz-Maya and F. Martín-Pliego. *Fundamentos de Inferencia Estadística.* Thomson - Paraninfo, 2005. Signatura: AL-519.23 RUI.

A. F. Trocóniz. *Probabilidades. Estadística. Muestreo.* Tebar-Flores, Madrid, 1987.