Lecture March, 11, 2013.

1. Let X be a random variable with the following probability distribution:

$P(x)$	$\frac{2}{10}$	$\frac{2-\theta}{10}$	$\frac{\theta}{10}$	$\frac{6}{10}$
x	1	2	3	4

   (a) What are the feasible values of θ?

   (b) Consider a random sample of independent observations from this distribution, whose values happen to be:

$$1, 3, 4, 4, 1, 2, 4, 4, 4, 4, 4, 4$$

   What is the MLE of θ?

2. Let X be distributed with density function $f_X(x;\theta) = (1+\theta)x^\theta$ for $0 \leq x \leq 1$ and $\theta > 1$. Obtain the MLE based on a sample of n independent observations.

3. A battery fires continuously shells on targets, all of them of the same size and at the same distance. A bored soldier wants to estimate the probability p that a shell hits its target. The following sequence of questions, that you must answer in his place, suggests one way of doing it.

   (a) What is the probability of hitting the target with the first shot? With the second (we assume that the first missed the target)? With the third? With the n-th shot?

   (b) If a given target, repeatedly fired upon, is hit with the 13th shot, what would be the maximum likelihood estimation of p?

   (c) After the first target is hit, the soldier witnesses the destruction of another two, of the same size and at the same distance, requiring respectively 22 and 14 shots. What would be the new MLE of p encompassing all information?

4. A random sample made of two independent observations from a $N(\alpha, 1)$ distribution is used to estimate α. The following two estimators ares considered:

$$\begin{aligned} \alpha^* &= \frac{2}{3}X_1 + \frac{1}{3}X_2 \\ \alpha^{**} &= \frac{2}{5}X_1 + \frac{4}{5}X_2 \end{aligned}$$

   (a) Which (if any) of those two estimators are unbiased?

   (b) What are their respective variances?

   (c) Can you obtain an unbiased estimator of α whose variance is less than that of α^*? If your answer is yes, provide an example; if your answer is no, explain why not.