Lecture January, 28, 2013

REVIEW:

- Distribution function, $F_X(x)$, density function $f_X(x)$, probability function $P_X(x)$.
- Expectation $E[g(X)]$, moments.
- Taylor expansion:

$$g(x) = g(a) + g'(a)(x-a) + \frac{1}{2!}g''(a)(x-a)^2 + \frac{1}{3!}g'''(a)(x-a)^3 + \ldots$$

- When $a = 0$ we have the Maclaurin expansion.
- A particular case we need:

$$e^t = 1 + t + \frac{1}{2!}t^2 + \frac{1}{3!}t^3 + \frac{1}{4!}t^4 + \ldots$$

- Characteristic,

$$\psi_X(u) = E[e^{iuX}]$$

and moment-generating,

$$\varphi_X(u) = E[e^{uX}]$$

functions. Relationship to moments.

- Convergence in probability, convergence in distribution ("in law") and the Central Limit Theorem (CLT).
- Newton's binomial formula:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

(Remember: $\binom{n}{x} = \frac{n!}{x!(n-x)!}$.)

BINARY DISTRIBUTION, $b(p)$:

- Simplest possible distribution for a two-state random variable.
- Probability function:

$$X = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{with probability } q = 1-p \end{cases}$$

- 0 and 1, conventional labels: could be heads/tails, success/failure, life/death, yes/no, etc.
- Quite easily $E[X] = p$, $\mathrm{Var}(X) = pq$. (What is the largest possible variance?)
- The characteristic function is $\psi(u) = E[e^{iuX}] = q + pe^{iu}$ (if you prefer the moment generating function, it would be $\varphi(u) = E[e^{uX}] = q + pe^{u}$).

Binomial distribution, $b(p, n)$:

- $X \sim B(p, n)$ if $X = X_1 + \ldots + X_n$ and X_i i.i.d as $b(p)$.
- Examples of common situations which can be described in terms of the binomial distribution.
- What is the probability function? Two ways:
  - Simple combinatorial argument: $P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$.
  - Use the characteristic (or moment-generating) function:

    $$\varphi_X(u) = \varphi_{X_1}(u) \times \cdots \times \varphi_{X_n}(u) = (q + pe^u)^n$$

    We have:

    $$\varphi_X(u) = P_X(0)e^0 + P_X(1)e^u + P_X(n)e^{nu} = (q + pe^u)^n$$

    and equating coefficients after applying Newton's binomial formula to the right hand side gives again the previous answer.
- If X and Y are both binomial, independent, with the same first parameter p and second parameter n_1 and n_2, the sum $X + Y \sim b(p, n_1 + n_2)$. Why?

Reading. For the review probability items, any text you may have used in the previous course, or if you want something in English [2]. For the binary and binomial distribution, [1] § 7.3 and 7.4, or [3], Chapter 25.

References

[1] J. Martín Pliego and L. Ruiz-Maya. *Estadística I : Probabilidad.* Ediciones AC, 2004. In the reserved collection, signature AL-519.2 MAR.

[2] Sheldon M. Ross. *A First Course In Probability.* Prentice Hall, 2003.

[3] A. Fz. Trocóniz. *Probabilidades. Estadística. Muestreo.* Tebar-Flores, Madrid, 1987.