eman ta zabal zazu

Universidad del País Vasco   Euskal Herriko Unibertsitatea

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Statistics Applied to Economics

Second Quiz, April, 27, 2012, Exam version: $\boxed{1}$

Last name: ______________________

First name: ______________________

DNI: ______________________

Group: ______________________

Instructor: ______________________

Section 1. Multiple choice questions

1. Consider a χ^2 goodness-of-fit test to a two-parameter distribution which is totally specified (the parameters are known). If we have k classes, the null hypothesis of fit will be rejected at the α significance level if the test statistic Z verifies:
   (a) $Z > {\chi^2}_{k-2+1,\ \alpha}$.
   (b) $Z > {\chi^2}_{k-2-1,\ \alpha}$.
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   (d) $Z > {\chi^2}_{k-1,\ \alpha}$.
   (e) All other answers are false.

2. You have two completely specifed hypothesis, H_0 y H_1. When using a hypothesis test of H_0 versus H_1, you might incur in two different kinds of error, whose probabilities are denoted by α and β. If the incorrect rejection of H_0 is very costly, and the non rejection of H_0 when H_1 is true has only a moderate cost, you would design a test: diseñarías un contraste:
   (a) With very small α
   (b) With very small β
   (c) With very small $1-\beta$
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   (e) Using Neyman-Pearson's theorem, which automatically chooses the optimal combination of α and β

3. What is the mean and variance of $\overline{X}$ obtained from a sample of n independent observations of a Poisson-distributed random variable $X \sim \mathcal{P}(\lambda)$?:
   (a) $\mathrm{E}(\overline{X})=\lambda$ and $\mathrm{Var}(\overline{X})=\dfrac{\lambda}{n}$.
   (b) $\mathrm{E}(\overline{X})=\mathrm{Var}(\overline{X})=\lambda$.
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4. The variance of an estimator is equal to its mean square error if:
   (a) The estimator is unbiased
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START OF A QUESTION BLOCK

Let X be a binary random variable with unknown parameter $p = P(X = 1)$. We have the following sample of independent values of X: $\{1, 1, 1, 0, 1\}$.

5. The maximum likelihood estimation of p is:
   (a) 0.8
   (b) 0.2
   (c) All other answers are false
   (d) 0.5
   (e) 0.1

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7. Let n_1 and n_2 be, respectively, the number of zeros and ones in an independent random sample of values of X. The maximum likelihood estimator of p is:
   (a) $\frac{n_2}{n_1+n_2}$
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   (d) $\frac{n_1+n_2}{n_2}$
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END OF A QUESTION BLOCK

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8. The capital city of Spain is:
   (a) Paris.
   (b) Pekín.
   (c) Madrid.
   (d) Kuala Lumpur.

9. You have two completely specified hypothesis, H_0 and H_1. The most powerful test of H_0 versus H_1 with $\alpha = 0.03$ has an associated β (probability of type II error) of 0.18. In order to obtain a new test (with the same sample size) giving a power of 0.85, we will have to:
   (a) Tolerate an increase of α.
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START OF A QUESTION BLOCK

Let X be a random variable normally distributed, $X \sim N(m, \sigma^2)$, of unknown mean and variance. Consider the following two estimators, based on a sample of independent values fo size n:

$$\hat{m}_1 = \frac{X_1 + X_2 + \cdots + X_n}{n-1}$$

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10. Which among $\hat{m}_1$ and $\hat{m}_2$ will be chosen, if our goal is to minimize $E(\hat{m} - m)^2$?
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12. It is thougth that the monthly number of units of a certain product consumed by a family follows the distribution specified in the table next:

x_i	0	1	2	3
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    (a) Is an unbiased estimator of the population mean, m
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14. We have two completely specified hypothesis, H_0 y H_1, and using the Neyman-Pearson's theorem we design the most powerful test of size $\alpha = 0.01$ for a sample of size n. When we use it:
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15. Let X be a random variable whose distributions depends on a unique parameter θ. We obtain the maximum likelihood estimation equating to zero:
    (a) The likelihood function
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Let X a random variable whose probability density function is:

$$f(x, \theta) = \theta x^{\theta}, \quad 0 \leq x \leq 1, \quad \theta > 0$$

We wish to test $H_0 : \theta = 1$ versus $H_1 : \theta = 0.5$ using a sample of size 1.

16. The highest power critical region of a given size is of the form:
    (a) $[C, 1]$
    (b) $[0, C]$
    (c) $(-\infty, 0]$
    (d) $[C_1, C_2]$ with $0 < C_1 < C_2 < 1$
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17. For a significance level of 5%, the most powerful critical region is:
    (a) $[0, 0.3162]$
    (b) $(-\infty, 0.5783]$
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18. The sample variance, $S^2 = \frac{\sum(x_i - \overline{x})^2}{n}$:
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20. Which of the following statements is true regarding the moments and maximum likelihood methods of estimation?
    (a) Both give interval estimates, and may give the same result
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21. Which, if any, of the following equalities is **always** true? (Note: MSE = "mean square error", ECM in Spanish.)
    (a) $\text{MSE}(\hat{\theta}) \leq \text{Var}(\hat{\theta})$
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    (a) Consistent
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Answers for the exam of type $\boxed{1}$

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Statistics Applied to Economics

Second Quiz, April, 27, 2012, Exam version: $\boxed{2}$

Last name: ____________________

First name: ____________________

DNI: ____________________

Group: ____________________

Instructor: ____________________

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$$f(x, \theta) = \theta x^{\theta}, \quad 0 \leq x \leq 1, \quad \theta > 0$$

We wish to test $H_0 : \theta = 1$ versus $H_1 : \theta = 0.5$ using a sample of size 1.

13. The highest power critical region of a given size is of the form:
    (a) $[C, 1]$
    (b) $[0, C]$
    (c) $(-\infty, 0]$
    (d) $[C_1, C_2]$ with $0 < C_1 < C_2 < 1$
    (e) All other answers are false

14. For a significance level of 5%, the most powerful critical region is:
    (a) $[0, 0.3162]$
    (b) $(-\infty, 0.5783]$
    (c) $[0, 0.5783]$
    (d) $[0.5783, +\infty)$
    (e) $[0.3162, +\infty)$

END OF A QUESTION BLOCK

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15. Which, if any, of the following equalities is **always** true? (Note: MSE = "mean square error", ECM in Spanish.)
    (a) $\text{MSE}(\hat{\theta}) \leq \text{Var}(\hat{\theta})$
    (b) $\text{MSE}(\hat{\theta}) = \text{Var}(\hat{\theta})$
    (c) $\text{MSE}(\hat{\theta}) \geq \text{Var}(\hat{\theta})$
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START OF A QUESTION BLOCK

Let X be a random variable normally distributed, $X \sim N(m, \sigma^2)$, of unknown mean and variance. Consider the following two estimators, based on a sample of independent values fo size n:

$$\hat{m}_1 = \frac{X_1 + X_2 + \cdots + X_n}{n - 1}$$

$$\hat{m}_2 = \frac{X_1 + X_2 + \cdots + X_n}{n + 1}$$

16. Which among $\hat{m}_1$ and $\hat{m}_2$ will be chosen, if our goal is to minimize $E(\hat{m} - m)^2$?
    (a) $\hat{m}_1$ because it has smaller variance than $\hat{m}_2$
    (b) $\hat{m}_2$ because it has smaller variance than $\hat{m}_1$
    (c) $\hat{m}_1$ because its mean squared error is smaller than that of $\hat{m}_2$
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17. Which of the following is true:
    (a) Both $\hat{m}_1$ and $\hat{m}_2$ are unbiased
    (b) $\hat{m}_1$ is asimptotically unbiased and $\hat{m}_2$ is asimptotically biased
    (c) Both $\hat{m}_1$ and $\hat{m}_2$ are asimptotically biased
    (d) Both $\hat{m}_1$ are $\hat{m}_2$ consistent
    (e) All other answers are false

END OF A QUESTION BLOCK

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18. You have two completely specifed hypothesis, H_0 y H_1. When using a hypothesis test of H_0 versus H_1, you might incur in two different kinds of error, whose probabilities are denoted by α and β. If the incorrect rejection of H_0 is very costly, and the non rejection of H_0 when H_1 is true has only a moderate cost, you would design a test: diseñarías un contraste:

    (a) With very small α

    (b) With very small β

    (c) With very small $1-\beta$

    (d) With $\alpha \approx 1$

    (e) Using Neyman-Pearson's theorem, which automatically chooses the optimal combination of α and β

19. An estimator $\hat{\theta}$ unbiased for θ, has variance $0.33/n$ (n is the sample size). It so happens that the Cramer-Rao lower bound for θ is $0.25/n$. Clearly, $\hat{\theta}$ is:

    (a) Consistent

    (b) Efficient

    (c) Of minimal MSE (mean square error)

    (d) Of minimal MSE (mean square error), but not efficient

    (e) Efficient, but not consistent

20. The sample variance, $S^2 = \frac{\sum(x_i-\overline{x})^2}{n}$:

    (a) Is a biased estimator of the population variance, σ^2

    (b) Is a unbiased estimator of the population variance, σ^2

    (c) All other answers are false

    (d) It is not an estimator

    (e) It is un unbiased estimator of the standard deviation, σ

21. We have two completely specified hypothesis, H_0 y H_1, and using the Neyman-Pearson's theorem we design the most powerful test of size $\alpha = 0.01$ for a sample of size n. When we use it:

    (a) The probability that H_0 is true is 0.01

    (b) The probability of rejecting H_0 when, in fact, it is true, is 0.01.

    (c) The power is 0.99.

    (d) The probability of selecting H_1 when, in fact, H_1 is true, is 0.99.

    (e) All other answers are false.

22. Which of the following things would be enough to know, in order to compute the α associated to a contrast of H_0 versus H_1 using a test statistic $S = S(\vec{X})$ and critical region C?

    (a) The value of β.

    (b) The distribution $F_X(x; H_1)$

    (c) The distribution of S under H_0.

    (d) The distribution of S under H_1.

    (e) All other answers are false.

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Universidad del País Vasco — Euskal Herriko Unibertsitatea

DIRECTIONS

1. Correctly answered questions give one point. There is only one correct answer to each question. Questions not correctly answered carry a penalty of -0.20 points, so it is better to leave a question unanswered rather than giving a wrong answer.

2. Our goal is to gauge your understanding and command of concepts learned during the course, not your visual sharpness. It is a fact, though, that in a multiple choice exam great attention has to be paid to the details. It is quite common that knowledgeable students waste their chances of a good grade because they do not pay sufficient attention to the precise wording of questions.

Please, do yourself a favour and read carefully before you answer!

3. It will probably help you to discard first answers that are clearly inadequate.

4. Students scoring 12 points or better in this exam will earn a full 0.75 points of their final grade.

5. The time scheduled for this exam is 1h.

Do not turn the page until advised to do so!
Mark the type of your exam on the orange sheet NOW!!

Answers for the exam of type $\boxed{2}$

Section 1. Multiple choice questions

1. Which of the following statements is true regarding the moments and maximum likelihood methods of estimation?
   - (a) Both give interval estimates, and may give the same result
   - **(b) Both produce point estimates, and may give the same result**
   - (c) Both produce point estimates, which are always different
   - (d) Both produce interval estimates, which are always different
   - (e) All other answers are false.

2. Consider a χ^2 goodness-of-fit test to a two-parameter distribution which is totally specified (the parameters are known). If we have k classes, the null hypothesis of fit will be rejected at the α significance level if the test statistic Z verifies:
   - (a) $Z > {\chi^2}_{k-2+1,\ \alpha}$.
   - (b) $Z > {\chi^2}_{k-2-1,\ \alpha}$.
   - (c) $Z > {\chi^2}_{k+1,\ \alpha}$.
   - **(d)** $Z > {\chi^2}_{k-1,\ \alpha}$**.**
   - (e) All other answers are false.

3. It is thougth that the monthly number of units of a certain product consumed by a family follows the distribution specified in the table next:

x_i	0	1	2	3
$P(x_i)$	θ	2θ	3θ	1-6θ

   A sample is taken providing 100 independent observations from such distribution. The sample mean and variance are 1.38 and 0.85. Using this information, the moment estimate of θ is:
   - (a) 1.38
   - (b) 0.15
   - **(c)** 0.162
   - (d) 0.85
   - (e) All other answers are false

4. Let X be a random variable whose distributions depends on a unique parameter θ. We obtain the maximum likelihood estimation equating to zero:
   - (a) The likelihood function
   - (b) The log of the likelihood function
   - **(c) The derivative of the log of the likelihood function with respect to θ**
   - (d) The derivative of the log of the likelihood function with respect to X

5. You have two completely specified hypothesis, H_0 and H_1. The most powerful test of H_0 versus H_1 with $\alpha = 0.03$ has an associated β (probability of type II error) of 0.18. In order to obtain a new test (with the same sample size) giving a power of 0.85, we will have to:

   (a) **Tolerate an increase of α.**

   (b) Tolerate a decrease of α.

   (c) Fix $\beta = 0.85$

   (d) Fix $\alpha + \beta = 0.85$

   (e) All other answers are false.

6. The sample mean, $\bar{X}$:

   (a) **Is an unbiased estimator of the population mean,** m

   (b) Is an biased estimator of the population mean, m

   (c) All other answers are false

   (d) It is not an estimator

   (e) It is un unbiased estimator of the standard deviation, σ

START OF A QUESTION BLOCK

Let X be a binary random variable with unknown parameter $p = P(X = 1)$. We have the following sample of independent values of X: $\{1, 1, 1, 0, 1\}$.

7. The maximum likelihood estimation of p is:

   (a) 0.8

   (b) 0.2

   (c) All other answers are false

   (d) 0.5

   (e) 0.1

8. The moment estimator of p is:

   (a) 0.8

   (b) 0.2

   (c) All other answers are false

   (d) 0.5

   (e) 0.1

9. Let n_1 and n_2 be, respectively, the number of zeros and ones in an independent random sample of values of X. The maximum likelihood estimator of p is:

   (a) $\frac{n_2}{n_1+n_2}$

   (b) $\frac{n_1}{n_1+n_2}$

   (c) $\frac{n_1+n_2}{n_1}$

   (d) $\frac{n_1+n_2}{n_2}$

   (e) All other answers are false

END OF A QUESTION BLOCK

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10. The capital city of Spain is:

   (a) Paris.

   (b) Pekín.

   (c) **Madrid.**

   (d) Kuala Lumpur.

11. What is the mean and variance of $\overline{X}$ obtained from a sample of n independent observations of a Poisson-distributed random variable $X \sim \mathcal{P}(\lambda)$?:

   (a) **E($\overline{X}$)=λ and Var($\overline{X}$)=$\frac{\lambda}{n}$.**

   (b) E($\overline{X}$)=Var($\overline{X}$)=λ.

   (c) E($\overline{X}$)=Var($\overline{X}$)=$\frac{\lambda}{n}$.

   (d) E($\overline{X}$)=Var($\overline{X}$)=$n\lambda$.

12. The variance of an estimator is equal to its mean square error if:

    (a) The estimator is unbiased

    (b) The estimator is efficient (and only in this case).

    (c) All other answers are false.

    (d) The variance is equal to the square of the bias of the estimator

    (e) The mean square error is equal to the square of the bias of the estimator

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