STATISTICS APPLIED TO BUSINESS ADMINISTRATION (BAD) - Second Year Academic Year 2023-24
STATISTICS APPLIED TO MARKETING (MD) - Second Year
STATISTICS APPLIED TO BUSINESS ADMINISTRATION (DD) - Third Year
First Call. May 28, 2024

INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle in which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. You can always cancel your mark by simply filling the rectangle below the answer you wish to cancel, and then fill the rectangle in which the new selected answer is located appropriately. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. **Every question correctly answered is worth 1 point, whereas each question incorrectly answered will not penalize your grade in any form. Therefore, you must answer all of the questions included in the exam**.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has six numbered sheets, going from 0.1 to 0.6. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet.
6. The maximum final grade for each of the parts of the exam (i.e., the multiple choice questions part and the exercises part) is 30 points. If you have participated in the on-going evaluation process, you will need to obtain 15 and 12 points in each part of the exam to pass it. Otherwise, 18 and 15 points in each part of the exam are required to pass it.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises.

MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 30 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 to 4 refer to the following exercise:

The waiting time for a patient (in minutes) at the only Health Center in a given town follows an exponential distribution with characteristic function given by $\psi_X(u) = (1 - 8iu)^{-1}$. It is established that this waiting time may be considered acceptable if it is no longer than 10 minutes. We assume independence in the waiting times for the different patients in that town. You need **to round up to one decimal place** the probability that the waiting time for a patient is acceptable.

2. If a random sample of 10 patients in that town is taken, the probability that at most 6 of them have an acceptable waiting time at the Health Center is:

   (A) 0.6496 (B) 0.3504 (C) 0.1503 (D) 0.8497 (E) 0.9894

3. In the same sample of 10 patients, the probability that exactly 4 patients **do not** have an acceptable waiting time at the Health Center is:

   (A) 0.6496 (B) 0.7999 (C) 0.2668 (D) 0.8497 (E) 0.2001

4. If we now take a random sample of 50 patients in that town, the approximate probability that at least 30 of them have an acceptable waiting time at the Health Center is:

   (A) 0.9554 (B) 0.6985 (C) 0.3015 (D) 0.3344 (E) 0.0446

Questions 5 and 6 refer to the following exercise:

Let Z be a r.v. having a binomial, $b(0.44, n)$, distribution with variance $\text{Var}(Z) = 2.9568$.

5. $P(Z = 4)$ is:

   (A) 0.7282 (B) 0.8206 (C) 0.1794 (D) 0.1015 (E) 0.2256

6. $P(Z \leq 1)$ is:

   (A) 0.0099 (B) 0.0050 (C) 0.0078 (D) 0.0378 (E) 0.0487

Questions 7 to 10 refer to the following exercise:

The number of clients arriving, **every twenty minutes**, at a shopping mall follows a Poisson distribution with $P(X = 5) = \frac{1}{2}P(X = 4)$. We assume independence between the arrivals of the different clients at the shopping mall.

7. The probability that, in a **forty-minute period**, at least 4 clients arrive at the shopping mall is:

   (A) 0.7350 (B) 0.4405 (C) 0.2650 (D) 0.5595 (E) 0.1247

8. The probability that, in a **one-hour period**, exactly 7 clients arrive at the shopping mall is:

   (A) 0.1545 (B) 0.1367 (C) 0.1892 (D) 0.2406 (E) 0.1465

9. The most likely number of clients expected to arrive at the shopping mall in a **two-hour period** is:

(A) 7 (B) 15 (C) 14 y 15 (D) 7 y 8 (E) 13 y 14

10. The approximate probability that, in a **four-hour period**, fewer than 25 clients arrive at the shopping mall is:

(A) 0.5714 (B) 0.4286 (C) 0.1587 (D) 0.8413 (E) 0.3421

Questions 11 and 12 refer to the following exercise:

Let X_1, X_2 and X_3 be three independent r.v., each having a $\gamma(a = 3, r = 5)$ distribution.

11. If we define the r.v. $Y = \frac{X_1+X_2+X_3}{2}$, the distribution of the r.v. Y is:

(A) $\gamma(6, 15)$ (B) $\gamma(\frac{2}{3}, 5)$ (C) $\gamma(6, 5)$ (D) $\gamma(\frac{2}{3}, 15)$ (E) $\gamma(\frac{3}{2}, 15)$

12. If we define the r.v. $V = 6X_2$, the value of k such that $P(k < V < 16) = 0.85$ is:

(A) 0.352 (B) 4.87 (C) 0.584 (D) 6.74 (E) 3.94

Questions 13 to 15 refer to the following exercise:

Let X, Y and Z be three independent r.v. such that: $X \in N(0, \sigma^2 = 4)$, $Y \in \gamma(\frac{1}{2}, 4)$ and $Z \in \gamma(\frac{1}{2}, \frac{8}{2})$, respectively.

13. If we define the r.v. $V_1 = \dfrac{2X^2}{Z}$, then $P(V_1 < 0.01683)$ is:

(A) 0.90 (B) 0.10 (C) 0.05 (D) 0.01 (E) 0.95

14. The value of k such that $P(X^2 > k) = 0.25$ is:

(A) 1.32 (B) 0.33 (C) 2.55 (D) 10.2 (E) 5.28

15. If we define the r.v. $V_2 = \dfrac{\sqrt{2}X}{\sqrt{Y}}$, then $P(-2.90 < V_2 < -1.40)$ is:

(A) 0.09 (B) 0.18 (C) 0.11 (D) 0.02 (E) 0.89

Questions 16 and 17 refer to the following exercise:

The four light bulbs in a given random sample have lifetimes equal to 4.1, 4.5, 3.9 and 5 thousand of hours, respectively. The lifetime (in thousands of hours) for the population of light bulbs (X) follows a probability distribution that depends on the parameters α and θ. It is known that $\mathrm{E}(X) = \alpha\ \theta$ and $\mathrm{E}(X^2) = \alpha\ \theta(1 + \alpha)$.

16. The method of moments estimate of α is approximately equal to:

(A) 9.13 (B) 6.82 (C) 4.82 (D) 3.42 (E) 1.54

17. The method of moments estimate of θ is approximately equal to:

(A) 1.63 (B) 1.28 (C) 0.27 (D) 0.20 (E) 0.50

Questions 18 to 20 refer to the following exercise:

Let Z be a r.v. such that $Z \in b(p,n)$. In order to estimate the parameter p, we propose the following two estimators:

$$T_1 = \frac{Z}{n}; \qquad T_2 = \frac{Z+1}{n+2}$$

18. We have that:

(A) Both estimators are unbiased (B) Only T_2 is unbiased (C) -
(D) Both estimators are biased (E) Only T_1 is unbiased

19. We have that:

(A) Only T_1 is consistent (B) Only T_2 is consistent (C) Both estimators are consistent
(D) - (E) -

20. The mean square error (MSE) for the estimator T_1, for $p = 1/2$, is:

(A) $\frac{1}{4}$ (B) $\frac{1}{4n}$ (C) $\frac{1}{2}$ (D) $\frac{4}{n}$ (E) $\frac{1}{2n}$

21. Let X be a r.v. having an exponential distribution with parameter $\frac{1}{\theta}$, $X \in \exp(\frac{1}{\theta})$. We consider the estimator $\hat{\theta} = \frac{X_1+X_2+X_3}{2k}$ for θ, obtained from a r.s. of size 3. What should the value of k be so that $\hat{\theta}$ is an unbiased estimator of θ?

(A) 3 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

Questions 22 and 23 refer to the following exercise:

Let X be a r.v. having a Poisson distribution with parameter λ. In order to test the null hypothesis $H_0 : \lambda = 2$ against the alternative hypothesis $H_1 : \lambda = 0.5$, a r.s. of size $n = 4$ has been taken, and $T = \sum_{i=1}^{4} X_i$ is used as test statistics.

22. At the $\alpha = 0.10$ significance level, the most powerful critical region rejects the null hypothesis if:

(A) $T \leq 4$ (B) $T \leq 5$ (C) $T \geq 5$ (D) $T \leq 3$ (E) $T \geq 4$

23. The probability of a Type II error for this significance level is:

(A) 0.0527 (B) 0.9834 (C) 0.9473 (D) 0.0166 (E) 0.1429

Questions 24 and 25 refer to the following exercise:

From a given variable that takes on values in the interval $(0,1)$, we wish to test the null hypothesis $H_0 : f(x) = 2x$ against the alternative hypothesis $H_1 : f(x) = 2 - 2x$. In order to do so, a random sample of a single element is taken, so that the test statistics X is considered.

24. The form of the most powerful critical region for this test is:

(A) $(0, C]$ (B) $[C, 1)$ (C) All false (D) $(C_1, C_2)^c$ (E) (C_1, C_2)

25. The specific form of the critical region for $\alpha = 0.05$ is:

(A) $(0, 0.328]$ (B) $[0.776, 1)$ (C) $(0, 0.224]$ (D) $(0.672, 0.776)^c$ (E) $[0.672, 1)$

Questions 26 and 27 refer to the following exercise:

Let $\overline{X}$ be the mean for a r.s. of size $n = 36$ taken from a population following a $N(m, \sigma^2 = 9)$ distribution. The decision rule to test $H_0 : m \leq 50$ against $H_1 : m > 50$ is to reject H_0 if $\overline{X} \geq 50.8$.

26. The significance level for this test is:

    (A) 0.9452 (B) 0.2981 (C) 0.4853 (D) 0.0548 (E) 0.7020

27. For $m = 50.8$, the power for this test is:

    (A) 0.9452 (B) 0.50 (C) 0 (D) 0.0548 (E) 0.7324

Questions 28 and 29 refer to the folllowing exercise:

A given firm devoted to growing peaches wishes to start selling its product in a new market. This initiative will be successful if the variance of the weight for the peaches is at most $50gr^2$, but it will be unsuccessful if it goes over that amount. In order to be able to make the best decision for the firm, a r.s. of size 10 has been taken, resulting in a sample variance of $s^2 = 53$. We assume normality in the distribution of the weight for the peaches.

28. The 95% confidence interval for the population variance:

    (A) $(27.89, 196.30)$ (B) $(38.92, 162.12)$ (C) $(43.37, 127.21)$ (D) $(57.34, 138.19)$ (E) $(31.36, 159.16)$

29. If we test $H_0 : \sigma^2 \leq 50gr^2$ against $H_1 : \sigma^2 > 50gr^2$, at the $\alpha = 0.05$ significance level, the firm will adopt the following decision:

    (A) Do not reject H_0 (B) - (C) Reject H_0 (D) - (E) -

30. We wish to test the hypothesis that the population's attitude with respect to a given product is the same for three different age intervals: (18 to 30, 31 to 45 and more than 45). In order to do so, a r.s. for each of the three age groups has been obtained. Sample sizes for the three samples has been 100, 150 and 80, respectively, and, for each of them, the percentage of people consuming the specific product regularly, sporadically or never was studied. The most appropriate test is:

    (A) Test of homogeneity

    (B) Test of difference of proportions

    (C) Test of equality of variances

    (D) χ^2 goodness-of-fit test to a completely specified distribution

    (E) Test of independence

EXERCISES (Time: 70 minutes)

A. (10 points, 20 minutes) Let $X_1, X_2, \ldots, X_n$ be independent r.v. such that, for each i, X_i follows a Poisson distribution with parameter $k_i\theta$. That is, $X_i \in \mathcal{P}(k_i\theta),\ i = 1, \ldots, n$, where the k_i's are known positive constants and $\theta > 0$. That is,

$$P(x_i) = \frac{k_i^{x_i}}{x_i!}\ e^{-k_i\theta}\ \theta^{x_i},\quad x_i = 0, 1, 2, \cdots,\quad k_i > 0,\ \theta > 0.$$

i) Show, **providing all relevant information**, that the maximum likelihood estimator of λ is:

$$\hat{\theta}_{\mathrm{ML}} = \frac{\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} k_i}$$

ii) Is it unbiased? Is it consistent? **Remark**: You can assume that $\lim_{n\to\infty} \sum_{i=1}^{n} k_i = +\infty$. In addition, **make sure that you provide all of the relevant information** to support your answers to these questions.

B. (10 points, 25 minutes) In a given Business and Economics School researchers are interested in testing if the level of satisfaction students have with respect to the classes they take is independent or not of the career choice they made based on their preferences (i.e., based on their vocation for a given career). In order to carry out this test, a random sample taken from the students going to class gave the following results:

	Yes vocation	Indiferent	No vocation	Totals
Satisfied	223	60	43	326
Indiferent	127	94	45	266
Not satisfied	40	23	36	99
Totals	390	177	124	691

Using this information, carry out the test of this hypothesis at a 5% significance level.

C. (10 points, 25 minutes)

Let X be a r.v. such that it follows an exponential distribution with mean $\frac{1}{\lambda}$. We wish to test the null hypothesis $H_0 : \lambda = 0.5$ against the alternative hypothesis $H_1 : \lambda = 1$. In order to do so, a random sample of size n, $X_1, X_2, \ldots, X_n$, has been taken.

i) Using the Neyman-Pearson theorem, and **providing all relevant details**, obtain the form of the most powerful critical region for this test.

ii) For an $\alpha = 0.05$ significance level and for a sample of size $n = 1$, obtain the specific critical region for this test.

iii) For an $\alpha = 0.05$ significance level and for a sample of size $n = 10$, obtain the specific critical region for this test.

Remark: You should recall the equivalences between the different distributions we studied in Chapter 3.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS (exam type 0)

1: C	11: A	21: B
2: B	12: E	22: A
3: E	13: B	23: A
4: A	14: E	24: A
5: C	15: A	25: C
6: A	16: D	26: D
7: A	17: B	27: B
8: E	18: E	28: A
9: C	19: C	29: A
10: C	20: B	30: A

SOLUTIONS TO EXERCISES

Exercise A

Given that $X_i \in \mathcal{P}(k_i\theta)$, we will have that

$$P(x_i) = \frac{k_i^{x_i}}{x_i!} \ e^{-k_i\theta} \ \theta^{x_i}, \quad x_i = 0, 1, 2, \cdots, \quad k_i > 0, \ \theta > 0.$$

i) In this way, the likelihood function will be given by

$$L(\theta) = P(x_1; k_1\theta) \cdots P(x_n; k_n\theta)$$

$$L(\theta) = \left[\frac{k_1^{x_1}}{x_1!} \ e^{-k_1\theta} \ \theta^{x_1}\right] \cdots \left[\frac{k_n^{x_n}}{x_n!} \ e^{-k_n\theta} \ \theta^{x_n}\right]$$

$$L(\theta) = \frac{\Pi_{i=1}^n k_i^{x_i}}{\Pi_{i=1}^n x_i!} \ e^{-\theta\sum_{i=1}^n k_i} \ \theta^{\sum_{i=1}^n x_i}$$

The maximum likelihood estimator of θ is the value that maximizes the likelihood function or, equivalently, its natural logarithm:

$$\ln L(\theta) = \sum_{i=1}^n x_i \ln k_i - \sum_{i=1}^n \ln x_i! - \theta \sum_{i=1}^n k_i + (\ln \theta) \sum_{i=1}^n x_i$$

If we take derivatives with respect to θ and equate this to zero, we will have that:

$$\frac{\partial \ln L(\theta)}{\partial \theta} = 0,$$

so that

$$-\sum_{i=1}^n k_i + \frac{\sum_{i=1}^n x_i}{\theta} = 0 \Longrightarrow \hat{\theta}_{\mathrm{ML}} = \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n k_i}$$

ii) This estimator will be unbiased if $\mathrm{E}(\hat{\theta}_{\mathrm{ML}}) = \theta$.

$$\mathrm{E}(\hat{\theta}_{\mathrm{ML}}) = \frac{1}{\sum_{i=1}^n k_i} \mathrm{E}\left(\sum_{i=1}^n X_i\right) = \frac{1}{\sum_{i=1}^n k_i} \sum_{i=1}^n \mathrm{E}(X_i) = \frac{1}{\sum_{i=1}^n k_i} \sum_{i=1}^n k_i\theta = \frac{\theta\left(\sum_{i=1}^n k_i\right)}{\left(\sum_{i=1}^n k_i\right)} = \theta$$

Therefore, $\hat{\theta}_{\mathrm{ML}}$ in an unbiased estimator of θ. To check if the estimator $\hat{\theta}_{\mathrm{ML}}$ is consistent, we have to verify if the two sufficient conditions stated below hold:

a) $\lim\limits_{n\to\infty} \mathrm{E}(\hat{\theta}_{\mathrm{ML}}) = \theta$

b) $\lim\limits_{n\to\infty} \mathrm{Var}(\hat{\theta}_{\mathrm{ML}}) = 0$

Given that $\hat{\theta}_{\mathrm{ML}}$ is an unbiased estimator of θ, condition a) holds. In addition, we have that

$$\lim_{n\to\infty} \mathrm{Var}(\hat{\theta}_{\mathrm{ML}}) = \lim_{n\to\infty} \frac{1}{\left(\sum_{i=1}^n k_i\right)^2} \sum_{i=1}^n \mathrm{Var}(X_i)$$

$$\lim_{n\to\infty} \operatorname{Var}(\hat{\theta}_{\mathrm{ML}}) = \lim_{n\to\infty} \frac{1}{\left(\sum_{i=1}^{n} k_i\right)^2} \sum_{i=1}^{n} k_i\theta = \lim_{n\to\infty} \frac{\left(\sum_{i=1}^{n} k_i\right)}{\left(\sum_{i=1}^{n} k_i\right)^2} \;\theta = \lim_{n\to\infty} \frac{\theta}{\sum_{i=1}^{n} k_i} = 0,$$

given that, according to the conditions provided in this exercise, we can assume that $\lim_{n\to\infty} \sum_{i=1}^{n} k_i = +\infty$. In this way, both of the sufficient conditions hold and, thus, $\hat{\theta}_{\mathrm{ML}}$ is a consistent estimator of θ.

Exercise B

The most appropriate test is a **test of independence**, so that the hypotheses to be tested are:

H_0: The variables satisfaction and vocation are independent

H_1: They are not independent

The probabilities $\hat{p}_{i\bullet}$ and $\hat{p}_{\bullet j}$ are estimated from the information provided by the sample. Therefore,

$$\hat{p}_{\mathrm{S},\bullet} = \frac{326}{691} \qquad \hat{p}_{\mathrm{IS},\bullet} = \frac{266}{691} \qquad \hat{p}_{\mathrm{NS},\bullet} = \frac{99}{691}$$

$$\hat{p}_{\bullet,\mathrm{YV}} = \frac{390}{691} \qquad \hat{p}_{\bullet,\mathrm{IV}} = \frac{177}{691} \qquad \hat{p}_{\bullet,\mathrm{NV}} = \frac{124}{691}$$

By using the available information, we build the following table:

	n_{ij}	$\hat{p}_{i,j} = \hat{p}_{i,\bullet} \cdot \hat{p}_{\bullet,j}$	$n\hat{p}_{i,j}$	$\frac{(n_{ij}-n\hat{p}_{i,j})^2}{n\hat{p}_{i,j}}$
S, YV	223	$326 \cdot 390/691^2$	183.994	8.269
S, IV	60	$326 \cdot 177/691^2$	83.505	6.616
S, NV	43	$326 \cdot 124/691^2$	58.500	4.107
IS, YV	127	$266 \cdot 390/691^2$	150.130	3.564
IS, IV	94	$266 \cdot 177/691^2$	68.136	9.818
IS, NV	45	$266 \cdot 124/691^2$	47.734	0.156
NS, YV	40	$99 \cdot 390/691^2$	55.875	4.511
NS, IV	23	$99 \cdot 177/691^2$	25.359	0.219
NS, NV	36	$99 \cdot 124/691^2$	17.765	18.716
	691	1	691	$z = 55.976$

Under the null hypothesis of independence, the test statistic $\sum_{i,j} \frac{(n_{ij}-n\hat{p}_{i,j})^2}{n\hat{p}_{i,j}} \sim \chi^2_{(I-1)(J-1)}$, where I is the number of categories in which the variable satisfaction has been divided ($I = 3$) and J is the number of categories in which the variable vocation has been divided ($J = 3$).

The decision rule indicates to reject the null hypothesis at the approximate 5% significance level if:

$$z > \chi^2_{(3-1)(3-1)\,,\,0.05} = \chi^2_{4,0.05}.$$

In this specific case:

$$z = 55.9766 > 9.49 = \chi^2_{4,0.05},$$

and, therefore, at the 5% significance level, we reject the null hypothesis of independence. That is, we can state that the level of satisfaction students have with respect to the classes they take is not statistically independent of the career choice they made based on their preferences (i.e., based on their vocation for a given career).

Exercise C

$$X \in \exp(\lambda) \quad \Rightarrow \quad f(x) = \lambda \mathrm{e}^{-\lambda x} \quad \text{si } x > 0, \quad \lambda > 0 \quad \Rightarrow \quad m = \frac{1}{\lambda}$$

$$\begin{array}{lllll} H_0 : \lambda = 0.5 & \Rightarrow & f(x) = 0.5\mathrm{e}^{-0.5x} & \text{if } x > 0 & \Rightarrow \quad m = 2 \\ H_0 : \lambda = 1 & \Rightarrow & f(x) = 1\mathrm{e}^{-1x} & \text{if } x > 0 & \Rightarrow \quad m = 1 \end{array}$$

i) In order to be able to obtain the form of the most powerful region for this test, we have to carry out the likelihood ratio test. That is,

$$\frac{L(\vec{x}; H_0)}{L(\vec{x}; H_1)} \le K \Longrightarrow \frac{\left[0.5\ e^{-0.5x_1}\right] \cdot \left[0.5\ e^{-0.5x_2}\right] \cdots \left[0.5\ e^{-0.5x_n}\right]}{\left[e^{-x_1}\right]\ \left[e^{-x_2}\right] \cdots \left[e^{-x_n}\right]} \le K$$

$$(0.5)^n\ e^{0.5\sum_{i=1}^n x_i} \le K \Longrightarrow e^{0.5\sum_{i=1}^n x_i} \le K_1 \Longrightarrow 0.5\sum_{i=1}^n x_i \le K_2 \Longrightarrow T = \sum_{i=1}^n x_i \le C$$

Therefore, the critical region for the test statistic, $T = \sum_{i=1}^n X_i$, will be of the form: $\mathrm{CR} = (0, C]$

ii) If $n = 1$, the test statistic is $T = \sum_{i=1}^1 X_i = X$, so that:

$$\alpha = 0.05 = P(\text{Reject } H_0 \mid H_0 \text{ is true }) = P\left[X \le C \mid X \in \exp(\lambda = 0.5)\right] = 1 - e^{-0.5C}$$

$$\Longrightarrow 0.95 = e^{-0.5C} \Longrightarrow \ln 0.95 = -0.5C \Longrightarrow C = 0.1026$$

Therefore, the critical region for this specific test will be: CR= $(0,\ 0.1026]$

iii) We should recall that:

$$X \in \exp(\lambda) \quad \equiv \quad X \in \gamma\,(a = \lambda,\ r = 1) \quad \Rightarrow \quad T = \sum_{i=1}^{10} X_i \in \gamma\,(a = \lambda,\ r = 10)$$

$$Y \in \gamma\left(\frac{1}{2},\ \frac{n}{2}\right) \quad \equiv \quad Y \in \chi^2_{\overline{n}|}$$

Under the null hypothesis, $\lambda = 0.5$, and, in addition, $n = 10$, so that:

$$X \in \exp(\lambda = 0.5) \quad \Rightarrow \quad T = \sum_{i=1}^{10} X_i \in \gamma\left(\frac{1}{2},\ \frac{20}{2}\right) \quad \equiv \quad Z \in \chi^2_{\overline{20}|}$$

Therefore,

$$\alpha = 0.05 = P(\text{Reject } H_0 \mid H_0 \text{ is true}) = P\left(T \le C \;\middle|\; T \in \chi^2_{\overline{20}|}\right)$$

$$\Longrightarrow C = \chi^2_{\overline{20}|0.95} = 10.9,$$

so that, the critical region for this specific test will be: CR= $(0,\ 10.9]$