STATISTICS APPLIED TO BUSINESS ADMINISTRATION (BAD) - Second Year Academic Year 2017-18
STATISTICS APPLIED TO MARKETING (MD) - Second Year
STATISTICS APPLIED TO BUSINESS ADMINISTRATION (DD) - Third Year
Second Call. June 28, 2018

INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has six numbered sheets, going from 0.1 to 0.6. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet.
6. The maximum final grade for each of the parts of the exam (i.e., the multiple choice questions part and the exercises part) is 30 points. You will need to obtain 15 points in each part of the exam to pass it.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises.

MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 20 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 to 4 refer to the following exercise:

The probability that a client visiting a given car dealer finally buys a car is 0.2. It is known that 10 clients visit this specific car dealer each day. We assume independence between the different clients.

2. The probability that, in a given day, more than four clients buy a car is:

   (A) 0.9672 (B) 0.1209 (C) 0.9936 (D) 0.8791 (E) 0.0328

3. If we assume independence between the distributions for different days, the approximate probability that, in a ten-day period, at most 25 clients buy a car is:

   (A) 0.9783 (B) 0.9162 (C) 0.8686 (D) 0.1314 (E) 0.0838

4. The probability that, in the above ten-day period, there is exactly one day in which the number of clients buying a car is larger than four is:

   (A) 0.2430 (B) 0.7570 (C) 0.1734 (D) 0.3280 (E) 0.4235

5. Let X be a r.v. with characteristic function $\psi_X(u) = (0.3 + 0.7e^{iu})^5$. The mean of this random variable is:

   (A) 0.70 (B) 0.21 (C) 0.30 (D) 1.50 (E) 3.50

Questions 6 to 8 refer to the following exercise:

Let $X_1, \ldots, X_5$ be five independent and identically distributed r.v. having a Poisson distribution with variance equal to 4.

6. We can state that:

   (A) $P(X_1 = 4) = P(X_1 = 5)$ (B) $P(X_1 = 4) = P(X_1 = 3)$ (C) $P(X_1 = 4) < P(X_1 = 3)$
   (D) $P(X_1 = 4) > P(X_1 = 3)$ (E) $P(X_1 = 4) < P(X_1 = 5)$

7. The probability $P(1 \leq X_1 \leq 5)$ is equal to:

   (A) 0.7851 (B) 0.7668 (C) 0.6935 (D) 0.6105 (E) 0.1563

8. If we define the r.v. $Y = \sum_{i=1}^{5} X_i$, then $P(Y < 23)$ is approximately equal to:

   (A) 0.6331 (B) 0.2877 (C) 0.5398 (D) 0.3669 (E) 0.7123

9. It is known that the mean of a r.v. having a $\gamma(a, 1)$ distribution is equal to es 4. We can then state that the value of a is equal to:

   (A) 4 (B) $\frac{1}{4}$ (C) 2 (D) $\frac{1}{2}$ (E) 16

Questions 10 and 11 refer to the following exercise:

Let X be a r.v. having a $\gamma(2,6)$ distribution.

10. The distribution of the r.v. $Y = 4X$ is:

(A) $\gamma(\frac{1}{2},6)$ (B) $\gamma(1,24)$ (C) $\gamma(2,24)$ (D) $\gamma(8,6)$ (E) $\gamma(2,6)$

11. The probability $P(Y \geq 21)$ is equal to:

(A) 0.05 (B) 0.90 (C) 0.95 (D) 0.75 (E) 0.10

Questions 12 to 14 refer to the following exercise:

Let $X_1,\ldots,X_5$ be independent r.v., each having a $N(1,\sigma^2=4)$ distribution.

12. If we define the r.v. $Z = \left(\frac{X_1-1}{2}\right)^2 + \left(\frac{X_2-1}{2}\right)^2 + \left(\frac{X_3-1}{2}\right)^2 + \left(\frac{X_4-1}{2}\right)^2$, we have that $P(Z > 9.49)$ is equal to:

(A) 0.10 (B) 0.95 (C) 0.75 (D) 0.90 (E) 0.05

13. If we define the r.v. $V = \dfrac{\left(\frac{X_5-1}{2}\right)}{\sqrt{\frac{Z}{4}}}$, we have that $P\left(-2.13 < V < 1.53\right)$ is equal to:

(A) 0.70 (B) 0.05 (C) 0.85 (D) 0.15 (E) 0.95

14. The probability $P\left(V^2 < 4.54\right)$ is equal to:

(A) 0.90 (B) 0.05 (C) 0.01 (D) 0.95 (E) 0.10

Questions 15 to 16 refer to the following exercise:

Let $X_1,\ldots,X_n$ be a r.s. taken from a population with probability mass function given by:

$$P(X=-1)=\theta, \quad P(X=0)=\theta, \quad P(X=1)=1-2\theta$$

In order to estimate the parameter θ, a r.s. of size $n=10$ has been taken, rendering the following results: -1, -1, -1, -1, 0, 0, 0, 0, 1, 1.

15. The method of moments estimate of θ is:

(A) 0.5 (B) 0.1 (C) 0.3 (D) 0.2 (E) 0.4

16. The maximum likelihood estimate of θ is:

(A) 0.5 (B) 0.1 (C) 0.2 (D) 0.3 (E) 0.4

Questions 17 to 18 refer to the following exercise:

Let X be a r.v. with probability density function given by:

$$f(x,\theta) = \begin{cases} \frac{2x}{\theta^2} & \text{for } 0 \leq x \leq \theta; \\ 0 & \text{otherwise} \end{cases}$$

It is known that the mean of this r.v. is $m = \frac{2\theta}{3}$. We wish to estimate the parameter θ and, in order to do so, a r.s. of size n, $X_1,\ldots,X_n$ has been taken.

17. The method of moments estimator of θ, $\hat{\theta}_{\text{MM}}$, will be:

(A) $\min\{X_i\}$ (B) $\overline{X}$ (C) $\max\{X_i\}$ (D) $\frac{2\overline{X}}{3}$ (E) $\frac{3\overline{X}}{2}$

18. The maximum likelihood estimator of θ, $\hat{\theta}_{\text{ML}}$, will be:

(A) $\max\{X_i\}$ (B) $\overline{X}$ (C) $\frac{3\overline{X}}{2}$ (D) $\frac{2\overline{X}}{3}$ (E) $\min\{X_i\}$

Questions 19 and 20 refer to the following exercise:

Let X be a r.v. with probability density function given by:

$$f(x,\theta) = \begin{cases} 2e^{2(\theta-x)} & \text{for } x \geq \theta,\ \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

It is known that the mean of this distribution is $m = \frac{1}{2} + \theta$ and its variance $\sigma^2 = \frac{1}{4}$. We wish to estimate the parameter θ and, in order to do so, a r.s. of size n, $X_1, \ldots, X_n$, has been taken, and the estimator $\hat{\theta} = \overline{X}$ is proposed.

19. The bias of the estimator $\hat{\theta}$ is:

(A) $\frac{\theta}{4}$ (B) $\frac{\theta}{2}$ (C) 0 (D) $\frac{1}{4}$ (E) $\frac{1}{2}$

20. The mean square error of the estimator $\hat{\theta}$ is:

(A) $\frac{1}{4}$ (B) $\frac{1}{4n} - \frac{1}{2}$ (C) $\frac{1}{4n} + \frac{1}{4}$ (D) $\frac{1}{4n} + \frac{1}{2}$ (E) $\frac{1}{4n}$

Questions 21 and 22 refer to the following exercise:

Let X be a r.v. with probability density function given by:

$$f(x,\theta) = \theta\, x^{\theta-1}, \quad x \in (0,1), \quad \theta > 0$$

In order to test the null hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 3$, a random sample of size $n = 1$ (i.e., X_1), is taken.

21. At the 5% significance level, the most powerful critical region for X_1 will be:

(A) $[0.475, 0.525)^{\text{C}}$ (B) $[0.475, 0.525)$ (C) $[0.975, 1)$ (D) $(0, 0.224]$ (E) $(0, 0.05]$

22. For the above significance level, the power for this test is, approximately equal to:

(A) 0.857 (B) 0.143 (C) 0.675 (D) 0.927 (E) 0.073

Questions 23 and 24 refer to the following exercise:

A given supermarket chain wishes to know its clients' monthly mean expense. In order to do so, a r.s. of 30 of its clients is taken, providing a mean expense of 65 euros. We assume that the clients' expense is normally distributed with variance $\sigma^2 = 225$ euros^2.

23. A 95% confidence interval for the mean expense is:

(A) $(58.91, 71.09)$ (B) $(63.47, 66.53)$ (C) $(59.63, 70.37)$ (D) $(60.54, 69.46)$ (E) $(61.84, 68.16)$

24. We wish to test the null hypothesis that the monthly mean expense is at least 70 euros, against the alternative hypothesis that it is smaller than this value. At the 5% significance level, the decision will be:

(A) Reject H_0 (B) - (C) - (D) - (E) Do not reject H_0

Questions 25 and 26 refer to the following exercise:

The number of cars that arrives at a given gas station each hour follows a Poisson distribution. We wish to test the null hypothesis that the mean of this distribution is at least 2, against the alternative hypothesis that it is smaller than 2. In order to do so, a r.s. of 4 hours, X_1, X_2, X_3, X_4, is taken,and the test statistic $Z = \sum_{i=1}^{4} X_i$ is considered.

25. At the 5% significance level, the most adequate decision will be to reject the null hypothesis if:

(A) $Z \leq 4$ (B) $Z \geq 3$ (C) $Z \leq 3$ (D) $Z \geq 13$ (E) $Z \geq 14$

26. Using the above decision rule, the probability of rejecting the null hypothesis when the mean is equal to 1 is:

(A) 0.4335 (B) 0.5665 (C) 0.7619 (D) 0.3772 (E) 0.6288

Questions 27 and 28 refer to the following exercise:

A given firm wishes to estimate its clients' mean expense when requesting some specific service. The estimation requires a 95% confidence level and an absolute error of 2 euros. It is known that the firm's total number of clients is 800, and that the variance of the clients' expense is equal to 300 euros^2.

27. What would be the minimum number of clients that need to be selected if simple random sampling with replacement is used?

(A) 289 (B) 213 (C) 202 (D) 162 (E) 385

28. What about if simple random sampling without replacement is used instead?

(A) 202 (B) 213 (C) 385 (D) 162 (E) 289

Questions 29 and 30 refer to the following exercise:

A given city wishes to estimate the mean household renting price. Authorities suspect that this value differs considerably between apartments located in the downtown area (zone A) and those in the suburbs (zone B). In addition, within each one of those zones, household renting prices are quite homogeneous. It is known that the number of households for rent in zone A is $N_A = 5000$, and in zone B is $N_B = 15000$, and that $\sigma_A^2 > \sigma_B^2$.

29. If, for a sample of size 500, stratified random sampling with proportional allocation is used, the corresponding sample sizes n_A and n_B will be:

(A) $n_A = 200$, $n_B = 300$ (B) $n_A = 300$, $n_B = 200$ (C) $n_A = 250$, $n_B = 250$
(D) $n_A = 125$, $n_B = 375$ (E) $n_A = 375$, $n_B = 125$

30. If optimal allocation is used, with the available information we can state that the corresponding sample sizes n_A and n_B will be such that:

(A) $n_A < 125$, $n_B > 375$ (B) $n_A > 300$, $n_B < 200$ (C) $n_A > 125$, $n_B < 375$
(D) $n_A > 375$, $n_B < 125$ (E) $n_A < 300$, $n_B > 200$

EXERCISES (Time: 75 minutes)

A. (10 points, 25 minutes)

A given refreshing drinks firm wishes to estimate the hourly mean energy expense its bottling machine has. In order to do so, a r.s. of 31 hours is taken, providing a mean of 20 euros and a variance of 25 euros^2. The electric company proposes the firm a rate change to reduce such expense. To be able to verify if the expense is smaller under this new rate, a new r.s. of 31 hours is taken under the new rate, providing a mean of 18 euros and a variance of 16 euros^2. We assume that the distribution of the hourly machine energy expense is normal, and that there is independence between the distributions of the expenses before and after the rate change and that, in addition, variances remain the same before and after the rate change.

i) Obtain a 95% confidence interval for the hourly mean energy expense before the rate change.

ii) Obtain a 95% confidence interval for the hourly mean energy expense difference before and after the rate change.

iii) The firm wishes to verify if the rate change has been really effective. In order to do so, it wishes to test the null hypothesis that the hourly mean energy expense has not changed, against the alternative hypothesis that the expense before the rate change was higher than that after it. At the 5% significance level, what will be the test decision? Please, do provide all relevant details to support your answer.

B. (10 points, 25 minutes)

Let X be a r.v. having a Poisson distribution with unknown parameter λ. In order to estimate this parameter, a r.s. of size n, $X_1, \dots, X_n$, has been taken, and two estimators have been proposed:

$$\hat{\lambda}_1 = \frac{X_2 + \cdots + X_{n-1}}{n-2} \qquad \text{y} \qquad \hat{\lambda}_2 = \frac{2X_1 + X_2 + \cdots + X_{n-1} + 2X_n}{n}$$

i) Compute the bias for both estimators. Are these estimators unbiased?

ii) Compute the variance for both estimators. Are these estimators consistent? Please, do provide all relevant details to support your answer.

iii) It is known that the Cramer-Rao lower bound for an unbiased estimator of λ is $\frac{\lambda}{n}$. Is any of the above estimators efficient? Please, do provide all relevant details to support your answer.

C. (10 points, 25 minutes)

Let X be a random variable with probability density function given by:

$$f(x, \theta) = \frac{1}{2} x^{\theta}, \quad x \in (0, 2), \quad \theta \in \{0, 1\}$$

We wish to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 0$. In order to do so, a random sample of one element ($n = 1$), X, is taken.

i) Obtain the form of the most powerful critical region for X.

ii) For a 4% significance level, what is the exact form of the critical region?

iii) For the above significance level, what is the power for this test?

iv) The interval $[0.3, 0.5]$ is proposed as a new critical region for X, where this new critical region has the same 4% significance level as the previous one. Without actually computing anything else, what will be the behavior of the power for this new critical region when compared to the previous one? Please, do provide all relevant details to support your answer.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS (exam type 0)

1: C	11: A	21: C
2: E	12: E	22: E
3: B	13: C	23: C
4: A	14: A	24: A
5: E	15: E	25: C
6: B	16: E	26: A
7: B	17: E	27: A
8: E	18: A	28: B
9: B	19: E	29: D
10: A	20: C	30: C

SOLUTIONS TO EXERCISES

Exercise A

$X \in N(m_1, \sigma_1^2)$ $\quad n_1 = 31 \quad \bar{x} = 20 \quad s_1^2 = 25$

$Y \in N(m_2, \sigma_2^2)$ $\quad n_2 = 31 \quad \bar{y} = 18 \quad s_2^2 = 16$

In addition, $\sigma_1^2 = \sigma_2^2$

i)

$$\text{CI}_{1-\alpha}(m_1) = \left(\overline{x} \pm t_{\overline{n-1}|\frac{\alpha}{2}} \frac{s_1}{\sqrt{n_1 - 1}} \right)$$

$$t_{\overline{30}|\frac{0.05}{2}} = 2.04$$

$$\text{CI}_{0.95}(m_1) = \left(20 \pm 2.04 \cdot \sqrt{\frac{25}{30}} \right) =$$

$$=(18.14\ ,\ 21.86)$$

ii)

$$\text{CI}_{1-\alpha}(m_1 - m_2) = \left(\overline{x} - \overline{y} \pm t_{\overline{n_1+n_2-2}|\frac{\alpha}{2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} \right)$$

$$t_{\overline{60}|\frac{0.05}{2}} = 2.00$$

$$\text{CI}_{0.95}(m_1 - m_2) = \left(20 - 18 \pm 2.00 \sqrt{\frac{1}{31} + \frac{1}{31}} \sqrt{\frac{31 \cdot 25 + 31 \cdot 16}{31 + 31 - 2}} \right) =$$

$$=(-0.34\ ,\ 4.34)$$

iii) We have to test:

$H_0 : m_1 = m_2 \quad \equiv \quad m_1 - m_2 = 0$

$H_1 : m_1 > m_2 \quad \equiv \quad m_1 - m_2 > 0$

Under H_0 :

$$\frac{\left(\bar{X} - \bar{Y}\right) - 0}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}} \in t_{\overline{n_1+n_2-2}|}$$

At the α significance level, the decision rule for this test is to reject H_0 if:

$$\frac{(\bar{x} - \bar{y}) - 0}{\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}} > t_{\overline{n_1+n_2-2}|\alpha}$$

In this case:

$$\frac{(20 - 18) - 0}{\sqrt{\frac{1}{31} + \frac{1}{31}} \sqrt{\frac{31 \cdot 25 + 31 \cdot 16}{31 + 31 - 2}}} = 1.71 \ > \ 1.67 = t_{\overline{60}|0.05}$$

Therefore, at the 5% significance level, the null hypothesis that the hourly mean energy expense has not changed before and after the rate change is rejected.

Exercise B

Given that X follows a Poisson distribution, we know that $\sigma^2 = m = E(X) = \lambda$.

i) The bias for any estimator is defined as $\mathrm{b}(\hat{\lambda}) = \mathrm{E}(\hat{\lambda}) - \lambda$.

To compute the bias for the proposed estimators, we compute their corresponding expected values, so that:

$$\mathrm{E}(\hat{\lambda}_1) = \mathrm{E}\left(\frac{X_2 + \cdots + X_{n-1}}{n-2}\right) = \left(\frac{\mathrm{E}(X_2) + \cdots + \mathrm{E}(X_{n-1})}{n-2}\right) = \left[\frac{(n-2)}{(n-2)}\right]\lambda = \lambda$$

$$\mathrm{E}(\hat{\lambda}_2) = \mathrm{E}\left(\frac{2X_1 + X_2 + \cdots + X_{n-1} + 2X_n}{n}\right) = \left[\frac{2\mathrm{E}(X_1) + \mathrm{E}(X_2) + \cdots + \mathrm{E}(X_{n-1}) + 2\mathrm{E}(X_n)}{n}\right] =$$

$$= \frac{4\lambda + (n-2)\lambda}{n} = \left[\frac{(n+2)}{n}\right]\lambda$$

Based on the above, the corresponding biases for these estimators will be:

$$\mathrm{b}(\hat{\lambda}_1) = \mathrm{E}(\hat{\lambda}_1) - \lambda = \lambda - \lambda = 0$$

$$\mathrm{b}(\hat{\lambda}_2) = \mathrm{E}(\hat{\lambda}_2) - \lambda = \frac{(n+2)}{n}\lambda - \lambda = \left(\frac{2}{n}\right)\lambda$$

An estimator is unbiased if its bias is equal to zero. Therefore, $\hat{\lambda}_1$ is an unbiased estimator of λ and $\hat{\lambda}_2$ is a biased one.

ii) We now compute the variances for the proposed estimators.

$$\mathrm{Var}(\hat{\lambda}_1) = \mathrm{Var}\left(\frac{X_2 + \cdots + X_{n-1}}{n-2}\right) = \frac{1}{(n-2)^2}\left[\mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_{n-1})\right] = \left[\frac{(n-2)}{(n-2)^2}\right]\mathrm{Var}(X) = \frac{\lambda}{n-2}$$

$$\mathrm{Var}(\hat{\lambda}_2) = \mathrm{Var}\left(\frac{2X_1 + X_2 + \cdots + X_{n-1} + 2X_n}{n}\right) = \frac{1}{n^2}\left[4\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \cdots + \mathrm{Var}(X_{n-1}) + 4\mathrm{Var}(X_n)\right] =$$

$$= \frac{8\lambda + (n-2)\lambda}{n^2} = \left[\frac{(n+6)}{n^2}\right]\lambda$$

Given that $\hat{\lambda}_1$ is unbiased and that its variance approaches zero as n goes to infinity, the sufficient conditions for consistency hold and, thus, we can state that $\hat{\lambda}_1$ is a consistent estimator for λ. With regard to $\hat{\lambda}_2$, we can verify that it is not unbiased, but it is asymptotically unbiased because:

$$\lim_{n\to\infty} \mathrm{E}(\hat{\lambda}_2) = \lim_{n\to\infty}\left[\frac{(n+2)}{n}\right]\lambda = \lambda$$

Therefore, in this case we have that, as $\hat{\lambda}_2$ is an asymptotically unbiased estimator whose variance approaches zero as n goes to infinity, the sufficient conditions for consistency also hold and, thus, $\hat{\lambda}_2$ is also a consistent estimator for λ.

iii) If we now first concentrate on the estimator $\hat{\lambda}_1$, we can verify that it is not efficient for λ because its variance ir larger than that of the Cramer-Rao lower bound. That is,

$$\mathrm{Var}(\hat{\lambda}_1) = \frac{\lambda}{n-2} > \frac{\lambda}{n}$$

As for the estimator $\hat{\lambda}_2$, we can also verify that, given that it is a biased estimator for λ, it cannot be an efficient estimator for λ. Therefore, none of the proposed estimators is efficient for λ.

Exercise C

The probability density function for the r.v. X is given by:

$$f(x, \theta) = \frac{1}{2}x^{\theta}, \quad x \in (0, 2), \quad \theta \in \{0, 1\},$$

and we wish to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 0$, based on the information provided from a random sample of one single element, X.

i) To obtain the form of the most powerful critical region for this test, we compute the likelihood ratio test. The corresponding likelihood functions under the null and alternative hypothesis will be given by:

$$L(x; \theta_0) = \tfrac{1}{2}x, \qquad \text{y} \qquad L(x; \theta_1) = \tfrac{1}{2},$$

Therefore, we have that:

$$\frac{L(x; \theta_0)}{L(x; \theta_1)} = \frac{\frac{1}{2}x}{\frac{1}{2}} \leq K, \; K > 0$$

$$\Longrightarrow x \leq C$$

That is, the form of the most powerful critical region for the test statistic X is $\text{CR} = (0, C]$.

ii) To obtain an $\alpha = 0.04$ confidence level, and taking into account that, under the null hypothesis, we have that:

$$f(x) = \frac{1}{2}x, \quad x \in (0, 2),$$

we must have that:

$$\alpha = 0.04 = P[X \in \text{CR}|H_0] = P[X \leq C|H_0] = \int_0^C \frac{1}{2}x dx = \frac{1}{4}\left[x^2\right]_0^C$$

$$\Longrightarrow \frac{1}{4}C^2 = 0.04 \Longrightarrow C^2 = 0.16 \Longrightarrow C = 0.4$$

That is, we reject the null hypothesis if $X \leq 0.4$.

iii) To compute the power for this test, we will have that:

$$\text{Power} = P[X \in \text{CR}|H_1] = P[X \leq 0.4|H_1] = \int_0^{0.4} \frac{1}{2}dx = \left[\frac{1}{2}x\right]_0^{0.4} = \frac{0.4}{2} = 0.2$$

iv) If a new critical region with the same significance level as the one in the previous items is proposed, we know that its power will be smaller than the most powerful critical region obtained in the previous items because, by its own definition, the critical region obtained by means of the likelihood ratio test is the most powerful critical region. Moreover, this fact is confirmed by the use of the Neyman-Pearson Theorem.