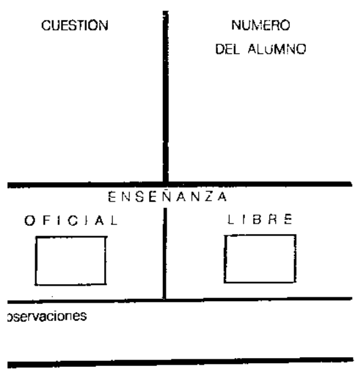
STATISTICS APPLIED TO BUSINESS ADMINISTRATION - Second Year Academic Year 2014-15
STATISTICS APPLIED TO MARKETING - Second Year
Second Call. July 8, 2015

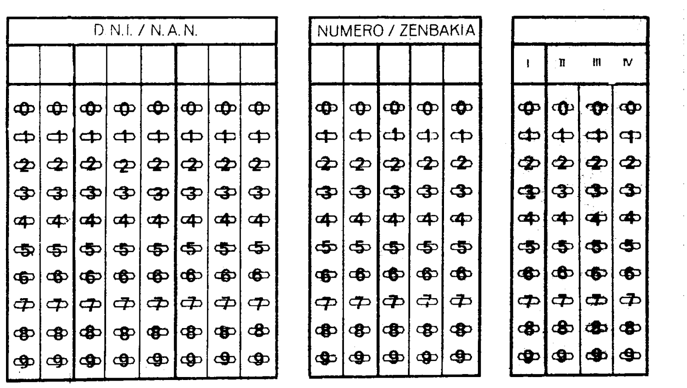
INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has six numbered sheets, going from 0.1 to 0.6. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as is illustrated in the example.
6. The maximum final grade for each of the parts of the exam is 30 points. You will need to obtain 15 points in each part of the exam to pass it.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises.

Example:

ID: $\underbrace{12345678}$ Exam type $\underbrace{0}$





MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 45 minutes)

1. **FREE-QUESTION**. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 to 4 refer to the following exercise:

The probability that a sportsman/sportswoman has an accident in a given competition is 0.08. We assume independence between the different sportmen/sportwomen.

2. If we take a r.s. of 10 sportmen/sportwomen, the probability that 4 of them have an accident is:

   (A) 0.0010 (B) 0.0025 (C) 0.9948 (D) 0.0052 (E) 0.6258

3. If we take a r.s. of 75 sportmen/sportwomen, the approximate probability that no more than 8 of them have an accident is:

   (A) 0.7439 (B) 0.1528 (C) 0.2560 (D) 0.8472 (E) 0.9318

4. If we take a r.s. of 600 sportmen/sportwomen, the approximate probability that more than 50 of them have an accident is:

   (A) 0.6480 (B) 0.5871 (C) 0.6985 (D) 0.3520 (E) 0.4129

Questions 5 to 7 refer to the following exercise:

Let X be a random variable with a Poisson distribution, which represents the number of weekly sales in a car sales dealer, so that the mean number of cars sold per week is 1.5.

5. The probability that, in a given week, 3 cars are sold is:

   (A) 0.0825 (B) 0.0972 (C) 0.1255 (D) 0.1572 (E) 0.1427

6. If we assume independence between the different weeks, the approximate probability that, in the 52 weeks of a given year, less than 52 cars are sold is:

   (A) 0.9987 (B) 0.4993 (C) 0.3670 (D) 0.6330 (E) 0.0014

7. The most likely number(s) of cars sold during the 52 weeks of a given year is (are):

   (A) only 78 (B) 77 and 78 (C) only 77 (D) only 79 (E) 78 and 79

8. Let X be a r.v. following a Poisson distribution with mean 13.2. We can state that:

   (A) $P(X = 3) > P(X = 2)$ (B) $P(X = 13) = P(X = 12)$ (C) $P(X = 23) > P(X = 22)$
   (D) $P(X = 14) > P(X = 13)$ (E) $P(X = 14) = P(X = 13)$

9. Let X_1 and X_2 be two independent r.v. such that $X_1 \in \gamma(a = 3, r = 1)$ and $X_2 \in \gamma(a = 4, r = 2)$. The distribution of the r.v. $3X_1 + 4X_2$ is:

   (A) $\gamma(1, 3)$ (B) $\gamma(3, 1)$ (C) $\gamma(3, 4)$ (D) $\gamma(7, 3)$ (E) $\gamma(25, 9)$

10. Let X, Y and Z be independent random variables with respective normal distributions such that: $X_1 \in N(1, \sigma^2 = 9)$, $X_2 \in N(3, \sigma^2 = 9)$ and $X_3 \in N(2, \sigma^2 = 4)$. Which one of the following random variables has a $t_{\overline{2}|}$ distribution?

(A) $\frac{X}{\sqrt{\frac{Y^2+Z^2}{2}}}$ (B) $\frac{X-1}{\left(\frac{Y-3}{3}\right)^2+\left(\frac{Z-2}{2}\right)^2}$ (C) All false (D) $\frac{\frac{X-1}{9}}{\sqrt{\frac{Y^2+Z^2}{2}}}$ (E) $\frac{\frac{X-1}{3}}{\sqrt{\frac{\left(\frac{Y-3}{3}\right)^2+\left(\frac{Z-2}{2}\right)^2}{2}}}$

Questions 11 to 13 refer to the following exercise:

Let X_1, X_2 and X_3 be independent r.v. such that their respective distributions are: $X_1 \in N(5, \sigma^2 = 9)$, $X_2 \in N(4, \sigma^2 = 4)$ and $X_3 \in N(3, \sigma^2 = 1)$.

11. If we define the r.v. $W = \left(\frac{X_1 - 5}{3}\right)^2 + \left(\frac{X_2 - 4}{2}\right)^2$, then $P(-4.61 < W < 7.38)$ is:

(A) 0.950 (B) 0.025 (C) 0.975 (D) 0.990 (E) 0.050

12. If we define the r.v. $Y = \frac{\sqrt{2}\,(X_3 - 3)}{\sqrt{\left(\frac{X_1-5}{3}\right)^2 + \left(\frac{X_2-4}{2}\right)^2}}$, then $P(Y > 2.92)$ is:

(A) 0.95 (B) 0.20 (C) 0.05 (D) 0.90 (E) 0.10

13. If we define the r.v. $Z = \frac{2\,(X_3 - 3)^2}{\left(\frac{X_1-5}{3}\right)^2 + \left(\frac{X_2-4}{2}\right)^2}$, then $P(Z \leq 8.53)$ is:

(A) 0.90 (B) 0.01 (C) 0.95 (D) 0.10 (E) 0.05

Questions 14 and 15 refer to the following exercise:

Let X be a r.v. such that its probability mass function is given by:

$$P(1) = \theta, \quad P(2) = 3\theta, \quad P(3) = 1 - 4\theta$$

In order to be able to estimate the parameter θ, a r.s. of size $n = 7$ has been taken providing the following results: 1, 1, 2, 2, 2, 3, 3.

14. The method of moment estimate of the parameter θ is:

(A) 0.20 (B) 0.10 (C) 0.08 (D) 0.15 (E) 0.05

15. The maximum likelihood estimate of the parameter θ is:

(A) $\frac{2}{28}$ (B) $\frac{5}{28}$ (C) $\frac{3}{28}$ (D) $\frac{6}{28}$ (E) $\frac{4}{28}$

Questions 16 to 18 refer to the following exercise:

Let X be a r.v. such that its probability density function is given by:

$$f(x;\theta) = \theta^2 x e^{-\theta x}, \quad x > 0$$

It is known that the mean of the r.v. X is $m = \frac{2}{\theta}$. In order to be able to estimate the parameter θ, a r.s. of size n, $X_1, X_2, \cdots, X_n$ has been taken.

16. The method of moments estimator of θ is:

(A) $\frac{2}{\overline{X}}$ (B) $2 + \overline{X}$ (C) $\frac{1}{\overline{X}}$ (D) $\overline{X}$ (E) $2\overline{X}$

17. The maximum likelihood estimator of θ is:

(A) $2\overline{X}$ (B) $2+\overline{X}$ (C) $\dfrac{2}{\overline{X}}$ (D) $\overline{X}$ (E) $\dfrac{1}{\overline{X}}$

18. We wish to test the null hypothesis $\theta = 4$ against the alternative hypothesis $\theta = 2$. The most powerful critical region for this test, based on the test statistic $Z = \sum_{i=1}^{n} X_i$, will be of the form:

(A) $Z \in (K_1, K_2)$ (B) $Z \leq K$ (C) All false (D) $Z \in (K_1, K_2)^c$ (E) $Z \geq K$

Questions 19 to 21 refer to the following exercise:

Let X_1, X_2, X_3, X_4 be a r.s. taken from a population having an exponential distribution with parameter $\frac{1}{\theta}$; that is, $f(x) = \frac{1}{\theta}e^{-\frac{1}{\theta}x}$ for $x \geq 0$. In order to be able to estimate the parameter θ, we propose the following two estimators:

$\hat{\theta}_1 = \frac{1}{6}(X_1 + X_2) + \frac{1}{3}(X_3 + X_4)$ and $\hat{\theta}_2 = \frac{X_1+X_2+X_3+X_4}{4}$

19. Which one of these estimators is unbiased?

(A) Only $\hat{\theta}_2$ (B) $\hat{\theta}_1$ and $\hat{\theta}_2$ (C) Only $\hat{\theta}_1$ (D) None of them (E) All false

20. Which one of these estimators has smaller variance?

(A) $\hat{\theta}_1$ (B) They have the same variance (C) All false (D) - (E) $\hat{\theta}_2$

21. Is $\hat{\theta}_1$ efficient?

(A) No (B) - (C) Yes (D) - (E) -

Questions 22 to 24 refer to the following exercise:

A given machine manufactures engines whose fuel consumption per hour follows an exponential distribution. Whenever the machine is working properly, the mean fuel consumption per hour is of 1 liter, and, if this is not the case, this mean fuel consumption per hour is of 2 liters. In order to be able to test if, in a given moment in time, the machine is working properly, one of those engines is taken and we decide to use the following decision rule: if the engine's fuel consumption in a give hour is greater than 1.2 liters, we reject the null hypothesis that the machine works properly, and we do not reject this hypothesis otherwise.

22. The significance level for this test is:

(A) 0.301 (B) 0.465 (C) 0.289 (D) 0.684 (E) 0.698

23. The power for this test is:

(A) 0.907 (B) 0.284 (C) 0.549 (D) 0.361 (E) 0.212

24. If we decide to use the alternative critical region given by $(0.163, 1.891)^c$, which has the same significance level the previously proposed unilateral test had, can we conclude that the aforementioned unilateral test is better than this bilateral test?

(A) No, because is has smaller power for the same significance level.

(B) No, because it has larger power for the same significance level.

(C) Yes, because it has smaller power for the same significance level.

(D) Yes, because it has larger power for the same significance level.

(E) All false

Questions 25 and 26 refer to the following exercise:

In a Poisson distribution setting, we wish to test the null hypothesis $H_0 : \lambda = 0.50$ against the alternative hypothesis $H_1 : \lambda = 2$. In order to do so, a r.s. of size $n = 2$, X_1 and X_2, has been taken, and we decide to use $Y = X_1 + X_2$ as the test statistic for this test.

25. At the 5% significance level, we reject the null hypothesis if the sample value of Y is **larger than or equal** to:

(A) 3 (B) 2 (C) 6 (D) 4 (E) 5

26. The power for this test is, approximately equal to:

(A) 0.37 (B) 0.61 (C) 0.91 (D) 0.57 (E) 0.76

Questions 27 and 28 refer to the following exercise:

We wish to test if the criminality index (measured as the ratio of the number of people who were robbed over the total number of people) in two large Spanish cities is the same. In order to do so, a r.s. of 1000 inhabitants was taken in each one of those cities providing that, in the first one of them, say city A, 32 people were robbed, whereas, in the other one, say city B, 24 were robbed.

27. The approximate 0.95 confidence interval for the difference of the proportions of people who were robbed, $p_A - p_B$, is:

(A) $(-0.004,\ 0.020)$ (B) $(0,\ 0.016)$ (C) $(-0.006,\ 0.022)$ (D) $(-0.01,\ 0.026)$ (E) $(-0.008,\ 0.024)$

28. We wish to test the null hypothesis that the criminality index for city A is no larger than that for city B. At the 5% significance level, the decision of the test will be:

(A) Reject the null hypothesis. (B) - (C) - (D) - (E) Do not reject the null hypothesis.

Questions 29 and 30 refer to the following exercise:

We wish to estimate the mean family transportation expenses in a given city. It has been observed that, depending on the city area (downtown or suburb) there are different family mean expenses, but that they are very homogeneous within each city area. Therefore, it has been decided to carry out a stratified sampling, asking a total of 200 families in the city for their expenses. It is known that the number of families in each of the two groups is: $N_1 = 6000$ in the downtown city area, and $N_2 = 4000$ in the suburbs city area. In addition, it is also known that, from a previous study, $\sigma_1^* = 10$ and $\sigma_2^* = 6$.

29. If we use proportional allocation, the sample sizes for the the two strata will be:

(A) $n_1 = 125,\ n_2 = 75$ (B) $n_1 = 120,\ n_2 = 80$ (C) $n_1 = 100,\ n_2 = 100$
(D) $n_1 = 57,\ n_2 = 143$ (E) $n_1 = 143,\ n_2 = 57$

30. If we use optimal allocation, the sample sizes for the two strata will be:

(A) $n_1 = 100,\ n_2 = 100$ (B) $n_1 = 57,\ n_2 = 143$ (C) $n_1 = 143,\ n_2 = 57$
(D) $n_1 = 120,\ n_2 = 80$ (E) $n_1 = 125,\ n_2 = 75$

EXERCISES (Time: 75 minutes)

A. (10 points, 25 minutes)

The following frequency table shows the number of accidents that occurred in a dangerous street crossing over a consecutive 100-day period

Number of accidents	0	1	2	≥ 3
Frequency	42	35	14	9

At the 5% significance level, test the hypothesis that the aforementioned number of accidents follows a Poisson distribution. For parameter estimation purposes, you can assume that the number of days in which 3 or more accidents occurred, you really had exactly 3 accidents occurring.

B. (10 points, 25 minutes)

We wish to obtain some insights about the daily sales volume a given store has, knowing that the daily sales volume is normally distributed with standard deviation of 500 thousand euros. Answer the following items:

i) What is the minimum sample size (i.e., number of days in which sales are accounted for) a s.r.s with replacement should have in order to obtain an estimation of the mean daily sales volume with a 95% confidence and an absolute error or precision no larger than 200 thousand euros?

ii) If a sample of 50 days, provided a mean of 3000 thousand euros, obtain the 95% confidence interval for the mean daily sales.

iii) A firm has 2 stores similar to the one above. That is, for both stores the daily sales volume follows a normal distribution with standard deviation of 500 thousand euros. A s.r.s. of 50 days is taken in each of those stores, providing means equal to 3000 and 3500 thousand euros, respectively. Can we accept the null hypothesis that both stores have the same mean daily sales? Assume a 5% significance level.

C. (10 points, 25 minutes)

Let X be a r.v. such that it follows an exponential distribution with mean $\frac{1}{\lambda}$. We wish to test the null hypothesis $H_0 : \lambda = 0.5$ against the alternative hypothesis $H_1 : \lambda = 1$. In order to do so, a random sample of size n, $X_1, X_2, \ldots, X_n$, has been taken.

i) Using the Neyman-Pearson theorem, and **providing all relevant details**, obtain the form of the most powerful critical region for this test.

ii) For an $\alpha = 0.05$ significance level and for a sample of size $n = 1$, obtain the specific critical region for this test.

iii) For an $\alpha = 0.05$ significance level and for a sample of size $n = 10$, obtain the specific critical region for this test.

Remark: You should recall the equivalences between the different distributions we studied in Chapter 3.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS (exam type 0)

1: C	11: C	21: A
2: D	12: C	22: A
3: D	13: A	23: C
4: D	14: A	24: D
5: C	15: B	25: D
6: E	16: A	26: D
7: B	17: C	27: C
8: A	18: E	28: E
9: A	19: B	29: B
10: E	20: E	30: C

SOLUTIONS TO EXERCISES

Exercise A

Given that we need to estimate the paramweter λ, this exercise corresponds to a goodness-of-fit test to a partially specified distribution.

$H_0 : X \in \mathcal{P}(\lambda)$, where λ is an unknown parameter that needs to be estimated from the available data information by the maximum likelihood method, and

$H_1 : X \notin \mathcal{P}(\lambda)$

$$\hat{\lambda}_{\mathrm{ML}} = \overline{x} = \frac{0 \cdot 42 + 1 \cdot 35 + 2 \cdot 14 + 3 \cdot 9}{100} = \frac{90}{100} = 0.9$$

The estimated theoretical probabilities $\hat{p}_i$ will be computed from either the available statistical tables or from the probability mass function given by:

$$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1 \ldots, \qquad \lambda > 0,$$

where we should recall that:

$$P(X = x) = \frac{\lambda}{x}\; P(X = x - 1)$$

In this way,

$P(X = 0) = \frac{e^{-0.9}\ 0.9^0}{0!} = e^{-0.9} = 0.4066$

$P(X = 1) = \frac{0.9}{1}\ P(X = 0) = 0.3659$

$P(X = 2) = \frac{0.9}{2}\ P(X = 1) = 0.1647$

$P(X \geq 3) = 1 - F(X = 2) = 1 - 0.9372 = 0.0628$

Therefore, we can build the table:

Class	n_i	$\hat{p}_i$	$n\hat{p}_i$	$\frac{(n_i - n\hat{p}_i)^2}{n\hat{p}_i}$
0	42	0.4066	40.66	0.0442
1	35	0.3659	36.59	0.0691
2	14	0.1647	16.47	0.3704
≥ 3	9	0.0628	6.28	1.1781
Total	$n = 100$	1	$n = 100$	$z = 1.6618$

In the table above, we can observe that the required conditions for this test hold. More specifically, we have that, for each one of the classes in the table, $n\hat{p}_i \geq 5$. Under the null hypothesis, the test statistic $\sum_i \frac{(n_i - n\hat{p}_i)^2}{n\hat{p}_i}$ converges to a $\chi^2_{(k-h-1)}$ distribution, where k is the number of classes in which the sample is divided(i.e., $k = 4$) and h is the number of estimated parameters (i.e., $h = 1$). At the approximate 5% significance level, the decision rule for this test is to reject the null hypothesis if:

$$z \geq \chi^2_{(4-1-1)\,,\,0.05} = \chi^2_{2,0.05}$$

In this case, we have that:

$$1.6618 < 5.99 = \chi^2_{(4-1-1)\,,\,0.05} = \chi^2_{2,0.05},$$

so that, at the approximate 5% significance level, we do not reject the null hypothesis that the number of accidents follows a Poisson distribution.

Exercise B

X : daily sales volume a given store has

$X \in N(m, \sigma = 500)$ (in thousands of euros)

i) The minimum sample size required under s.r.s with replacement is:

$$n = \frac{t^2_{\frac{\alpha}{2}} \sigma^2}{\delta^2}$$

In this case, we have that: (1) 95% confidence level $\Rightarrow \alpha = 0.05 \Rightarrow t_{\frac{\alpha}{2}} = 1.96$; and (2) absolute error or precision: $\delta \leq 200$, so that:

$$n = \frac{t^2_{\frac{\alpha}{2}} \sigma^2}{\delta^2}$$

$$n = \left(\frac{1.96 \cdot 500}{200}\right)^2$$

$$n = 24.01$$

Therefore, the sample size should be $n \geq 25$.

ii) $n = 50, \quad \bar{x} = 3000$

$$\mathrm{CI}_{1-\alpha}(m) = \left(\overline{x} \pm t_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

$$\mathrm{CI}_{0.95}(m) = \left(3000 \pm 1.96 \cdot \frac{500}{\sqrt{50}}\right)$$

$$=(3000 \pm 138.59)$$

$$=(2861.41,\ 3138.59)$$

iii) We have to test the hypotheses:

$H_0 : m_1 = m_2 \quad \equiv \quad m_1 - m_2 = 0$

$H_1 : m_1 \neq m_2 \quad \equiv \quad m_1 - m_2 \neq 0$

Under H_0 :

$$\frac{(\bar{X} - \bar{Y}) - 0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \in N(0, 1)$$

In this case, the data available for the two stores are as follows:

$n_1 = 50 \quad \bar{x}_1 = 3000 \quad \sigma_1 = 500$

$n_2 = 50 \quad \bar{x}_1 = 3500 \quad \sigma_2 = 500$

$$\left|\frac{(3000 - 3500) - 0}{\sqrt{\frac{500^2}{50} + \frac{500^2}{50}}}\right| = 5 \geq 1.96 = t_{0.05/2}$$

Therefore, at the 5% significance level, we reject the null hypothesis that both stores have the same mean daily sales.

Exercise C

$$X \in \exp(\lambda) \qquad \Rightarrow \qquad f(x) = \lambda e^{-\lambda x} \quad \text{si } x > 0, \qquad \lambda > 0 \qquad \Rightarrow \qquad m = \frac{1}{\lambda}$$

$$H_0 : \lambda = 0.5 \qquad \Rightarrow \qquad f(x) = 0.5e^{-0.5x} \quad \text{if } x > 0 \qquad \Rightarrow \qquad m = 2$$
$$H_0 : \lambda = 1 \qquad \Rightarrow \qquad f(x) = 1e^{-1x} \quad \text{if } x > 0 \qquad \Rightarrow \qquad m = 1$$

i) In order to be able to obtain the form of the most powerful region for this test, we have to carry out the likelihood ratio test. That is,

$$\frac{L(\vec{x}; H_0)}{L(\vec{x}; H_1)} \leq k$$
$$\frac{0.5\ e^{-0.5x_1} \cdot 0.5\ e^{-0.5x_2} \ \cdots \ 0.5\ e^{-0.5x_n}}{e^{-x_1} \ \cdot \ e^{-x_2} \ \cdots \ e^{-x_n}} \leq k$$
$$(0.5)^n \ e^{0.5 \sum_{i=1}^n x_i} \leq k$$
$$e^{0.5 \sum_{i=1}^n x_i} \leq k'$$
$$0.5 \sum_{i=1}^n x_i \leq k''$$
$$Z = \sum_{i=1}^n x_i \leq C$$

Therefore, the critical region for the test statistic, $Z = \sum_{i=1}^n X_i$, will be of the form: $\quad \text{CR} = (0, C]$

ii) If $n = 1$, the test statistic is $Z = \sum_{i=1}^1 X_i = X$, so that

$$\begin{aligned} \alpha = 0.05 =& P(\text{reject } H_0 \mid H_0 \text{ is true }) = \\ =& P(X \leq C \mid X \in \exp(\lambda = 0.5)) = \\ =& 1 - e^{-0.5C} \\ 0.95 =& e^{-0.5C} \qquad \Rightarrow \qquad \ln 0.95 = -0.5C \qquad \Rightarrow \qquad C = 0.1026 \end{aligned}$$

Therefore, the critical region for this specific test will be: CR= $(0,\ 0.1026]$

iii) We should recall that:

$$X \in \exp(\lambda) \qquad \equiv \qquad X \in \gamma\,(a = \lambda,\ r = 1) \qquad \Rightarrow \qquad Z = \sum_{i=1}^{10} X_i \in \gamma\,(a = \lambda,\ r = 10)$$
$$Y \in \gamma\left(\frac{1}{2},\ \frac{n}{2}\right) \qquad \equiv \qquad Y \in \chi^2_{n|}$$

Under the null hypothesis, $\lambda = 0.5$, and, in addition, $n = 10$, so that:

$$X \in \exp(\lambda = 0.5) \qquad \Rightarrow \qquad Z = \sum_{i=1}^{10} X_i \in \gamma\left(\frac{1}{2},\ \frac{20}{2}\right) \qquad \equiv \qquad Z \in \chi^2_{20|}$$

$$\begin{aligned} \alpha = 0.05 =& P(\text{reject } H_0 \ \mid H_0 \text{ is true}) = \\ =& P\left(Z = \sum_{i=1}^{10} X_i \leq C \ \mid X_i \in \exp(\lambda = 0.5) \right) = \\ =& P\left(Z = \sum_{i=1}^{10} X_i \leq C \ \middle|\ Z \in \chi^2_{20|} \right) \qquad \Rightarrow \qquad C = \chi^2_{20|0.95} = 10.9 \end{aligned}$$

Therefore, the critical region for this specific test will be: CR= $(0,\ 10.9]$