The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with.

To select an answer, all you need to do is mark in the orange code sheet, filling the rectangle over which the selected answer is located appropriately. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advise you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.

In the multiple choice questions part of the exam there is always only one correct answer for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.

The exam has four numbered sheets, going from 0.1 to 0.5. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.

The maximum final grade is 20 points.

Please fill in your personal information in the appropriate places in the code sheet.

Example: 12545 PEREZ, Ernesto Exam type 0 Resit
MULTIPLE CHOICE QUESTIONS (Time: 60 minutes)

1. FREE-QUESTION. The capital of Spain is:
   (A) Paris  (B) Sebastopol  (C) Madrid  (D) London  (E) Pekin

Questions 2 and 3 refer to the following exercise:

The random variable $X$ follows a distribution such that its probability mass function is given by:

\[ P(X = -1) = 3\theta^3 \quad P(X = 0) = 1 - 6\theta^3 \quad P(X = 1) = 3\theta^3 \]

In order to test the null hypothesis $H_0 : \theta = 0.50$ against the alternative hypothesis $H_1 : \theta = 0.20$, a r.s. of size $n = 1$ has been taken, so that the rejection rule indicates that we should reject the null hypothesis if $X = 0$.

2. The significance level for this test is:
   (A) 0.375  (B) 0.024  (C) 0.625  (D) 0.250  (E) 0.750

3. The probability of a type II error for this test is:
   (A) 0.048  (B) 0.952  (C) 0.375  (D) 0.250  (E) 0.750

Questions 4 to 5 refer to the following exercise:

Let $X$ be a r.v. with probability density function given by:

\[ f(x, \theta) = 3^\theta x^{\theta-1}e^{-3x} \quad x > 0, \quad \theta > 0 \]

In order to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$, a random sample of size $n = 1$ has been taken.

4. At a given significance, the most powerful critical region for the test statistic $X$ is of the form:
   (A) $X \leq C$  (B) $X \in (C_1, C_2)$  (C) All false  (D) $X \in (C_1, C_2)$  (E) $X \geq C$

5. At the $\alpha = 5\%$ significance level, the null hypothesis $H_0$ is rejected if:
   (A) $X \geq 0.9986$  (B) $X \in (0.0171, 0.998)$  (C) $X \leq 0.9986$  (D) $X \leq 0.0171$  (E) $X \geq 0.0171$

Questions 6 to 7 refer to the following exercise:

The number of clients that go to a specific branch of a bank every hour follows a Poisson distribution. The director of the branch considers that s/he should open a new bank counter for the public if the average number of people going to this specific branch every hour is of at least 7. In order to test for the need to open a new bank counter; that is, $H_0 : \lambda \geq 7$, against the alternative hypothesis $H_1 : \lambda < 7$, s/he has information on the number of people going to this specific branch for one specific hour.

6. At the $\alpha = 10\%$ significance level, the most powerful decision rule for this test will be to reject $H_0$ if the number of clients going to this specific branch every hour is:
   (A) $X \geq 3$  (B) $X \geq 4$  (C) $X \leq 7$  (D) $X \leq 4$  (E) $X \leq 3$

- 0.2 -
7. For this test and for $\lambda = 5$, the probability of a type II error will be:

(A) 0.8753  (B) 0.2650  (C) 0.1247  (D) 0.7350  (E) 0.1404

Questions 8 and 9 refer to the following exercise:

An individual is interested in buying a specific type of DVD reader. Before doing so, s/he asks for its price at 31 stores, obtaining a sample mean price of 105 euros, with a sample standard deviation of 20 euros. We assume normality.

8. With a 90% confidence level, we can state that the mean price for this specific DVD reader is in the interval:

(A) (97.55, 112.45)  (B) (100.22, 109.78)  (C) (99.25, 110.75)

(D) (98.79, 111.21)  (E) (101.35, 108.65)

9. With a 90% confidence level, we can state that the variance of the price for this specific DVD reader is in the interval:

(A) (307.69, 601.94)  (B) (14.16, 33.51)  (C) (283.11, 670.27)

(D) (255.64, 706.34)  (E) (326.44, 585.45)

Questions 10 and 11 refer to the following exercise:

Let $X$ and $Y$ be two independent r.v., so that its corresponding distributions are: $X \in N(m_X, \sigma_X^2 = 25)$ and $Y \in N(m_Y, \sigma_Y^2 = 36)$, respectively. In order to test $H_0 : m_X = m_Y$ against $H_1 : m_X \neq m_Y$, two r.s. of sizes $n_X = 30$ and $n_Y = 30$, one in each population, have been taken, so that $\bar{x} = 82$ and $\bar{y} = 80$.

10. A 90% confidence interval for $(m_X - m_Y)$ is, approximately:

(A) $(-0.3386, 4.3386)$  (B) $(-1.3347, 5.3347)$  (C) $(0.7949, 4.7949)$

(D) $(0.3386, 4.3386)$  (E) $(-0.7949, 4.7949)$

11. At the $\alpha = 10\%$ significance level, the decision of the test will be:

(A) Do not reject $H_0$  (B) -  (C) Reject $H_0$  (D) -  (E) -

Questions 12 and 13 refer to the following exercise:

A researcher in the School of Biology at the University of the Basque Country is interested in estimating the proportion of rats that, having being exposed to a given risk factor, will develop lung cancer. The researcher randomly selects 150 rats among those that have been exposed to the risk factor over the last few years and for which the required information on the possible development of lung cancer was available. Data indicated that, among the selected 150 rats, 57 of them developed lung cancer.

12. A 95% confidence interval for the proportion of rats that, having been exposed to the risk factor, would develop lung cancer is, approximately:

(A) $(0.34, 0.42)$  (B) $(0.32, 0.44)$  (C) $(0.25, 0.51)$

(D) $(0.28, 0.48)$  (E) $(0.30, 0.46)$
13. At the 5% significance level, the researcher wishes to test the null hypothesis that the proportion of rats exposed to the risk factor that will develop lung cancer is larger than or equal to 0.40. The decision of the test will be:

(A) Do not reject the null hypothesis  
(B) It cannot be decided  
(C) Reject the null hypothesis  
(D) -  
(E) -

Questions 14 and 15 refer to the following exercise:

We wish to test if the type of health care provided to a specific patient in an emergency situation is related to the neighborhood the patient usually lives in. In order to do so, a r.s. is taken and patients are classified according to the neighborhood they usually live in (urban or rural neighborhood) and to the type of health care provided in an emergency situation (home health care, neighborhood health-service hospital – ambulatorio-, or referral to the closest available hospital).

14. The most adequate type of test for carrying out the aforementioned test is:

(A) Homogeneity  
(B) Independence  
(C) Comparison of means  
(D) Goodness-of-fit to a completely specified distribution  
(E) Goodness-of-fit to a partially specified distribution

15. The number of degrees of freedom for the distribution of the test statistic for the aforementioned test will be:

(A) 3  
(B) 2  
(C) 6  
(D) 5  
(E) 4

Questions 16 and 17 refer to the following exercise:

It is known that pharmaceutical expenses in a given population follow a normal distribution, and it is suspected that that variance of these expenses for individuals younger than 50 years old, $\sigma^2_1$, is greater than or equal than the variance of these expenses for individuals at least 50 years old, $\sigma^2_2$. In order to test $H_0 : \sigma^2_1 \geq \sigma^2_2$ against $H_1 : \sigma^2_1 < \sigma^2_2$, two random samples, each of size 31, are taken in each one of the two populations, providing $s^2_1 = 62$ and $s^2_2 = 65$.

16. A 98% confidence interval for the ratio of variances, $\sigma^2_1/\sigma^2_2$ is:

(A) (0.592, 1.536)  
(B) (0.399, 2.280)  
(C) (0.399, 1.536)  
(D) (0.592, 1.755)  
(E) (0.518, 1.755)

17. At the $\alpha = 1\%$ significance level, the decision of the test will be:

(A) Do not reject $H_0$  
(B) -  
(C) Reject $H_0$  
(D) -  
(E) -

Questions 18 and 20 refer to the following exercise:

We have two different coins and it is assumed that the probability of obtaining heads, $p$, is the same for both coins. In order to test this hypothesis, the first coin is tossed $n_1 = 250$ times, obtaining 200 heads, and the second coin is also tossed $n_2 = 400$ times, obtaining 300 heads.

18. Under the null hypothesis, What is the estimated value of the probability $p$?

(A) 0.2308  
(B) 0.7692  
(C) All false  
(D) 0.9889  
(E) 0.0111

- 0.4 -
19. If we label by $Z_1$ and $Z_2$, respectively, the number of heads obtained after tossing each one of the coins, the most appropriate test statistic for the aforementioned test, as well as its approximate distribution, under $H_0$, will be:

(A) $\frac{Z_1}{n_1} - \frac{Z_2}{n_2} \in N(0, \sigma^2 = 0.0011)$  
(B) All false  
(C) $\frac{Z_1}{n_1} - \frac{Z_2}{n_2} \in N(0, \sigma^2 = 0.0003)$  
(D) $\frac{Z_1}{n_1} - \frac{Z_2}{n_2} \in N \left( \frac{1}{4}, \sigma^2 = 0.1775 \left( \frac{1}{250} + \frac{1}{400} \right) \right)$  
(E) $Z_1 - Z_2 \in b(p = \frac{1}{4}, n = 65)$

20. At the 10% significance level, the decision of the test will be:

(A) Do not reject $H_0$  
(B) -  
(C) -  
(D) -  
(E) Reject $H_0$

SOLUTIONS

1: C  11: A  
2: D  12: E  
3: A  13: A  
4: E  14: B  
5: A  15: B  
6: E  16: B  
7: D  17: A  
8: D  18: B  
9: C  19: A  
10: A  20: A