

INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.
3. In the multiple choice questions part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.20 points. Questions that have not been answered do not penalize your grade in any form.
4. The exam has four numbered sheets, going from 0.1 to 0.4. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.
5. The maximum final grade is 20 points
7. Please fill in your personal information in the appropriate places in the code sheet.

Exam type	<u>0</u>	Resit
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QUESTION	NUMERO DEL ALUMNO	
ENSEÑANZA		
OFICIAL		LIBRE
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Observaciones		

[illegible]

NUMERO / ZENBAKIA				
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
3	3	3	3	3
4	4	4	4	4
5	5	5	5	5
6	6	6	6	6
7	7	7	7	7
8	8	8	8	8
9	9	9	9	9

	I	II	III	IV
1				
2				
3				
4				
5				
6				
7				
8				
9				

MULTIPLE CHOICE QUESTIONS (Time: 60 minutes)

1. FREE-QUESTION. The capital of Spain is:

(A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 to 4 refer to the following exercise:

In a canned asparagus line of production, it is known that the probability that a given can is defective is equal to 0.08. We assume independence between the different cans in the line of production.

2. If 10 asparagus cans are produced in this line of production, what is the probability that none of them is defective?

(A) 0.7360 (B) 0.2640 (C) 0.5656 (D) 0.4344 (E) 0.0736

3. If 60 asparagus cans are produced in this line of production, what is the approximate probability of having exactly 3 defective cans?

(A) 0.15 (B) 0.14 (C) 0.85 (D) 0.27 (E) 0.20

4. If we now have that 300 asparagus cans are produced in this line of production, what is the approximate probability of having more than 30 defective cans?

(A) 0.916 (B) 0.881 (C) 0.200 (D) 0.899 (E) 0.084

Questions 5 and 6 refer to the following exercise:

The number of daily workers' leaves in a given firm follows a Poisson distribution with parameter $\lambda = 4$. We assume independence between the distribution of leaves for different days.

5. The probability that, in a two-day period, there are more than 6 workers' leaves is:

(A) 0.3134 (B) 0.8088 (C) 0.1912 (D) 0.6866 (E) 0.4529

6. The approximate probability that, in a ten-day period, there are at most 31 workers' leaves is:

(A) 0.9099 (B) 0.9222 (C) 0.4840 (D) 0.0778 (E) 0.0901

7. Let X be a random variable having a Student t distribution with 5 degrees of freedom. The value of k such that $P(X > k) = 0.2$ holds is equal to:

(A) -1.48 (B) 0.92 (C) -0.92 (D) 1.48 (E) 2.02

8. Let X_1 and X_2 be two independent r.v. having a $\gamma(1, 2)$ and a $\gamma(2, 1)$ distribution, respectively. If we define the r.v. $Y = 2X_1 + 4X_2$, then the distribution of the r.v. Y is:

(A) $\gamma(0.5, 3)$ (B) χ_3^2 (C) $\gamma(0.25, 3)$ (D) χ_{12}^2 (E) $\gamma(2, 3)$

9. Let X be a r.v. having a $\gamma(\frac{5}{10}, 1)$ distribution. The probability that X takes on values larger than 4.61 is:

(A) 0.90 (B) 0.05 (C) 0.10 (D) 0.95 (E) 0.50

10. Let X be an exponential r.v., so that its variance is equal to 0.25. The cumulative distribution function of X evaluated at $x = 0.5$ is equal to:

(A) 0.865 (B) 0.632 (C) 0.117 (D) 0.368 (E) 0.135

Questions 11 and 12 refer to the following exercise:

Let X , Y and Z be three independent random variables with respective distributions: $N(5, \sigma^2 = 9)$, $N(4, \sigma^2 = 4)$ and $N(0, 1)$.

11. Which of the random variables listed below follows a χ_3^2 distribution?

(A) $\frac{X^2-5}{9} + \frac{Y^2-4}{4} + Z^2$ (B) $\frac{X^2-5}{3} + \frac{Y^2-4}{2} + Z^2$ (C) $X^2 + Y^2 + Z^2$
 (D) $\left(\frac{X-5}{3}\right)^2 + \left(\frac{Y-4}{2}\right)^2 + Z^2$ (E) All false

12. Which one of the random variables listed below follows a t_2 distribution?

(A) $\frac{X}{\sqrt{\frac{Y^2+Z^2}{2}}}$ (B) $\frac{X-5}{\left(\frac{Y-4}{2}\right)^2+Z^2}$ (C) All false (D) $\frac{\frac{X-5}{9}}{\sqrt{\frac{Y^2+Z^2}{2}}}$ (E) $\frac{\frac{X-5}{3}}{\sqrt{\left(\frac{Y-4}{2}\right)^2+Z^2}}$

Questions 13 and 14 refer to the following exercise:

Let X be a r.v. with probability mass function given by:

$$P(x; \theta) = \frac{e^{-\theta} \theta^{x-1}}{(x-1)!}, \quad x = 1, 2, \dots$$

It is known that the mean of this random variable is $m = \theta + 1$.

In order to estimate the parameter θ , a r.s. of size n , X_1, X_2, \dots, X_n , has been taken.

13. The method of moments estimator of θ is:

(A) $\bar{X} - 1$ (B) $\bar{X} + 1$ (C) \bar{X} (D) $\frac{1}{\bar{X}}$ (E) $\frac{1}{\bar{X}} - 1$

14. The maximum likelihood estimator of θ is:

(A) \bar{X} (B) $\bar{X} + 1$ (C) $\frac{1}{\bar{X}} - 1$ (D) $\frac{1}{\bar{X}}$ (E) $\bar{X} - 1$

Questions 15 and 16 refer to the following exercise:

Let X be a r.v. with probability mass function given by:

$$P(X = 1) = \theta; \quad P(X = 2) = 2\theta; \quad P(X = 3) = 1 - 3\theta$$

In order to obtain an estimate of the parameter θ , a r.s. of size $n = 10$ has been taken, providing the following results: 1, 1, 1, 1, 2, 2, 3, 3, 3, 3.

15. The method of moments estimate of θ is:

(A) 0.15 (B) 0.20 (C) 0.25 (D) 0.10 (E) 0.30

16. The maximum likelihood estimate of θ is:

(A) 0.25 (B) 0.20 (C) 0.30 (D) 0.10 (E) 0.15

Questions 17 to 20 refer to the following exercise:

Let X be a r.v. having $N(m, \sigma^2 = 9)$ distribution. We wish to estimate its mean m . In order to do so, a r.s. of size n , X_1, X_2, \dots, X_n , has been taken, and the estimator $\hat{m} = \frac{2(X_2 + X_4 + \dots + X_n)}{n}$ is proposed. We assume that n is an even number, and a multiple of two.

17. Is \hat{m} an unbiased estimator of m ?

- (A) No (B) - (C) Yes (D) - (E) -

18. The variance of \hat{m} is equal to:

- (A) $\frac{9}{n}$ (B) $\frac{18}{n^2}$ (C) All false (D) $\frac{9}{2n}$ (E) $\frac{18}{n}$

19. If the Cramer-Rao lower bound for a regular and unbiased estimator of m is $\frac{9}{n}$, the efficiency of \hat{m} is equal to:

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) 1 (D) $\frac{9}{n}$ (E) 2

20. Is \hat{m} and efficient estimator of m ?

- (A) Yes (B) - (C) - (D) - (E) No

SOLUTIONS

1: C 11: D

2: D 12: E

3: A 13: A

4: E 14: E

5: D 15: C

6: E 16: B

7: B 17: C

8: A 18: E

9: C 19: A

10: B 20: E