INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with.

2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.

3. In the multiple choice questions part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.20 points. Questions that have not been answered do not penalize your grade in any form.

4. The exam has three numbered sheets, going from 0.1 to 0.3. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.

5. The maximum final grade is 15 points

6. Please fill in your personal information in the appropriate places in the code sheet.

Example:

| 12545 | PEREZ, Ernesto | Exam type 0 | Resit |
MULTIPLE CHOICE QUESTIONS (Time: 40 minutes)

1. FREE-QUESTION. The capital of Spain is:
   (A) Paris  (B) Sebastopol  (C) Madrid  (D) London  (E) Pekin

Questions 2 to 4 refer to the following exercise:
We consider a series of births, where we keep the records about the gender of the newborns. In a given population, it is known that 51% of newborns are boys.

2. The probability that, in a series of 20 births, exactly 5 of the newborns are boys is:
   (A) 0.25  (B) 0.0345  (C) 0.082  (D) 0.75  (E) 0.012

3. If we now consider a series of 200 births, which one of the following statements is true with regard to the distribution of the total number of newborns in the aforementioned series?
   (A) Both the mean and the variance of the number of newborns that are boys are equal to 102 boys
   (B) Approximately, we have that the mean and standard deviation of the number of newborns that are boys are 50 and 7 boys, respectively
   (C) Both the mean and the variance of the number of newborns that are boys are approximately equal to 50 boys
   (D) The mean and standard deviation of the number of newborns that are boys are 102 and 7 boys, respectively
   (E) All false

4. The probability that, in a series of 200 births, more than 60% of the newborns are boys is, approximately:
   (A) 0.6443  (B) 0.0044  (C) 0.9821  (D) 0.3557  (E) 0.9956

5. Let $X$ be a r.v. following a binomial distribution, such that $X \sim b(p = 0.8, n = 10)$. The probability that the r.v. $X$ takes on values no larger than 3 is:
   (A) $\binom{10}{3} 0.8^3 0.2^7$  (B) 0.0009  (C) $\binom{10}{3} 0.8^3 0.2^7$  (D) 0.9999  (E) 0.80

6. A public transportation company is interested in the study of the distribution of the number of breakdowns its busses suffer. They have verified that the mean daily number of buses that suffers breakdowns is of two busses. We assume that the number of breakdowns busses suffer follows a Poisson distribution and, in addition, we assume independence between the number of breakdowns occurring in different days. The approximate probability that, in a given month of 30 days, the number of breakdowns busses suffer is larger than 40 is:
   (A) $\Phi(2.52)$  (B) $\Phi(0.33)$  (C) $\Phi(0)$  (D) $1 - \Phi(0.33)$  (E) $1 - \Phi(2.52)$

7. Let $X$ be a r.v. following a Poisson distribution with variance equal to 4.5. For this r.v., we have that:
   (A) $P(X = 4) < P(X = 5)$
   (B) $P(X = 4) > P(X = 5)$
   (C) $P(X = 2) > P(X = 3)$
   (D) $P(X = 4) = P(X = 5)$
   (E) All false
8. Let $X$ be a r.v. with characteristic function given by $\psi_X(u) = \left(1 - \frac{iu}{(1^2)}\right)^{-3/2}$. For this r.v., we have that:

(A) The r.v. $X$ follows a $\gamma\left(\frac{3}{2}, \frac{3}{2}\right)$ distribution

(B) The variance of the r.v. $X$ is equal to 3

(C) The mean of the r.v. $X$ is equal to $1/2$

(D) The r.v. $X$ follows a $\chi^2$ distribution with 3 degrees of freedom

(E) The distribution of the r.v. $X$ is unknown

9. Let $X_i$ be independent and identically distributed random variables following a $\gamma\left(\frac{1}{2}, \frac{3}{2}\right)$ distribution, for $i = 1, \ldots, 10$. If we define the r.v. $Z$ as the sum of the aforementioned variables, then we have that the distribution of the r.v. $Z$ will be:

(A) $\gamma\left(\frac{1}{2}, 30\right)$

(B) $\gamma\left(\frac{10}{2}, 30\right)$

(C) $\chi^2$ with 15 degrees of freedom

(D) Unknown

(E) $\chi^2$ with 30 degrees of freedom

10. Let $X$ be a r.v. following an $F_{8,10}$ distribution. The value for $k$ such that $P(X > k) = 0.95$ holds is:

(A) 0.2985

(B) 0.3257

(C) 5.060

(D) 3.070

(E) 3.350

Questions 11 and 12 refer to the following exercise:

Let $X$ be a r.v. following an exponential distribution with mean equal to $1/2$.

11. The variance of the r.v. $X$ is:

(A) $\frac{1}{4}$

(B) $\frac{1}{4}$

(C) 2

(D) 4

(E) 8

12. $P(-1 < X < 3)$ is:

(A) 0.1328

(B) 0.0025

(C) 0.2231

(D) 0.7768

(E) 0.9975

13. Let $X$ and $Y$ be two independent random variables such that $X \in \chi^2_1$ and $Y \in \chi^2_5$. If we define the r.v. $Z = \frac{\sqrt{X}}{\sqrt{Y}}$, then $P(Z > 2.02)$ is:

(A) 0.05

(B) 0.90

(C) 0.10

(D) 0.95

(E) 0.75

Questions 14 and 15 refer to the following exercise:

Let $X_1$ and $X_2$ be two independent r.v. such that $X_1 \in N(4, \sigma^2 = 9)$ and $X_2 \in N(0, \sigma^2 = 9)$. We define the r.v. $V = \frac{1}{9} [(X_1 - 4)^2 + X_2^2]$.

14. The distribution of the r.v. $V$ will be:

(A) $\chi^2_2$

(B) $[N(0, 1)]^2$

(C) $F_{1,1}$

(D) $t_2$

(E) Unknown

15. $P(V < 5.99)$ is:

(A) 0.95

(B) 0.10

(C) 0.05

(D) 0.02

(E) 0.75
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