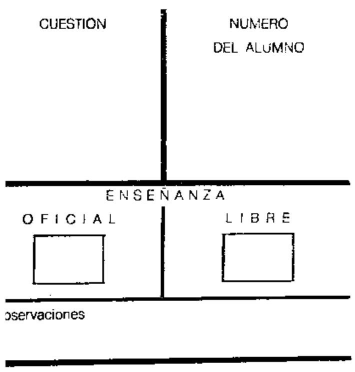
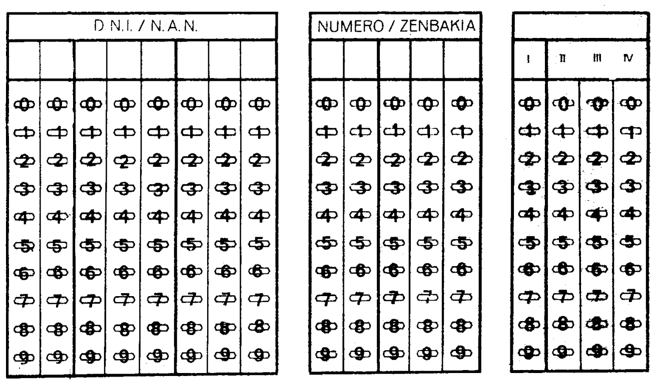
INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.
3. In the multiple choice questions part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will not penalize your grade at all. Questions that have not been answered do not penalize your grade in any form.
4. The exam has four numbered sheets, going from 0.1 to 0.4. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.
5. The maximum final grade is 15 points
7. Please fill in your personal information in the appropriate places in the code sheet.

Example:

$\underbrace{12545}$ PEREZ, Ernesto Exam type $\underbrace{0}$ Resit





MULTIPLE CHOICE QUESTIONS (Time: 45 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 and 3 refer to the following exercise:

Let X be a r.v. with probability density function given by:

$$f(x,\theta) = \begin{cases} \left(\frac{1}{2\theta}+1\right) x^{-\left(\frac{1}{2\theta}+2\right)} & \text{for } x \geq 1,\ \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

In addition, it is known that the mean of the r.v. X is $m = (2\theta + 1)$.

In order to estimate the parameter θ, a r.s. of size n, $X_1, \ldots, X_n$, has been taken.

2. The method of moments estimator of the parameter θ, $\hat{\theta}_{MM}$, will be:

   (A) $\frac{\overline{X}-1}{2}$ (B) $\frac{1}{2\overline{X}}$ (C) $\overline{X}$ (D) $2\overline{X}-1$ (E) $\frac{2}{\overline{X}-1}$

3. Is this an unbiased estimator of θ?

   (A) No (B) - (C) Yes (D) - (E) -

4. Let X be a r.v. with probability density function given by:

$$f(x,\theta) = \begin{cases} \frac{2x}{\theta^2}\, e^{-\left(\frac{x}{\theta}\right)^2} & \text{for } x \geq 0,\ \theta > 0; \\ 0 & \text{otherwise,} \end{cases}$$

   In order to estimate the parameter θ, a r.s. of size n, $X_1, \ldots, X_n$ has been taken. The maximum likelihood estimator of the parameter θ, $\hat{\theta}_{ML}$, will be:

   (A) $\overline{X}$ (B) $\frac{n}{\sum_{i=1}^n X_i^2}$ (C) $\left(\frac{n}{\sum_{i=1}^n X_i^2}\right)^{1/2}$ (D) $\left(\frac{\sum_{i=1}^n X_i^2}{n}\right)^{1/2}$ (E) $\frac{\sum_{i=1}^n X_i^2}{n}$

Questions 5 to 8 refer to the following exercise:

Let $X_1, X_2, \ldots, X_n$ be a r.s. from a population with probability mass function given by:

$$P(X=-1) = P(X=1) = \frac{\theta}{2}, \quad P(X=0) = 1-\theta$$

5. The method of moments estimator of the parameter θ is:

   (A) $\frac{\sum_{i=1}^n X_i^2}{n}$ (B) $\frac{\sum_{i=1}^n X_i^2}{2n}$ (C) $2\overline{X}$ (D) $\frac{2\sum_{i=1}^n X_i^2}{n}$ (E) $\frac{\overline{X}}{2}$

6. Is this an unbiased estimator of θ?

   (A) Yes (B) - (C) - (D) - (E) No

7. In order to obtain an estimate of the parameter θ, a r.s. has been taken, providing the following results: -1, -1, -1, -1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1. A method of moments estimate of θ is:

   (A) $\frac{2}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) $\frac{3}{4}$

8. For the same sample in the previous question, a maximum likelihood estimate of θ is:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{3}{4}$ (E) $\frac{2}{3}$

Questions 9 and 10 refer to the following exercise:

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population having a $N(m, \sigma^2)$ distribution, with $\sigma^2 > 0$. Let us consider the following estimators for the parameter m:

$$\hat{m}_1 = \frac{X_1 + X_2 + \cdots + X_n}{(n+1)}, \quad \hat{m}_2 = \frac{X_1 + X_2 + \cdots + X_n}{n} = \overline{X}$$

9. For these estimators we have that:
   (A) Both estimators are unbiased
   (B) $\hat{m}_1$ is unbiased and $\hat{m}_2$ is biased
   (C) Both estimators are biased
   (D) $\hat{m}_1$ is biased and $\hat{m}_2$ is unbiased
   (E) It cannot be determined from the information provided

10. For these estimators we have that:
   (A) None of these estimators is consistent
   (B) Both estimators are consistent
   (C) Only $\hat{m}_2$ is consistent
   (D) Only $\hat{m}_1$ is consistent
   (E) It cannot be determined from the information provided

Questions 11 and 12 refer to the following exercise:

Let X be a r.v. with probability density function given by:

$$f(x, \theta) = (\theta + 1)x^{\theta}, \quad 0 \leq x \leq 1, \quad \theta > 0$$

We wish to test the null hypothesis $H_0 : \theta = 2$ against the alternative hypothesis $H_1 : \theta = 1$. In order to do this, a r.s. of size $n = 1$, X_1, has been taken.

11. The most powerful critical region for X_1 is of the form:

(A) $[0, C]$ (B) $[C_1, C_2]^C$ (C) $[C, 1]$ (D) $[C_1, C_2]$ (E) All false

12. At the $\alpha = 0.05$ significance level, the most powerful critical region for this test will be:

(A) $[0.3684, 1]$ (B) $[0.3684, 0.6316]^C$ (C) All false
(D) $[0.3684, 0.6316]$ (E) $[0, 0.3684]$

Questions 13 to 15 refer to the following exercise:

Let X be a r.v. with probability mass function given by:

$$P(X=-2)=\frac{\theta}{2}, \qquad P(X=0)=\frac{3\theta}{2}, \qquad P(X=2)=1-2\theta$$

We wish to test the null hypothesis $H_0 : \theta = 0.10$ against the alternative hypothesis $H_1 : \theta = 0.30$. In order to do this, a r.s. of size $n=1$, X, is taken and the null hypothesis will be rejected if $X \in \{-2, 0\}$; that is, if the resulting value for X is either -2 or 0.

13. The significance level for this test is:

(A) 0.60 (B) 0.20 (C) 0.15 (D) 0.40 (E) 0.80

14. The probability of type II error for this test is:

(A) 0.15 (B) 0.20 (C) 0.80 (D) 0.40 (E) 0.60

15. The power for this test is:

(A) 0.40 (B) 0.15 (C) 0.20 (D) 0.60 (E) 0.80