## INSTRUCTIONS

- 1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with.
- 2. To select an answer, all you need to do is mark in the orange code sheet, filling the rectangle over which the selected answer is located appropriately. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.
- 3. In the multiple choice questions part of the exam there is always only one correct answer for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will not penalize your grade at all. Questions that have not been answered do not penalize your grade in any form.
- 4. The exam has four numbered sheets, going from 0.1 to 0.4. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.
- 5. The maximum final grade is 15 points
- 7. Please fill in your personal information in the appropriate places in the code sheet.

Example:

PEREZ, Ernesto 12545

Exam type 0 Resit



D. N.I. Z N. A. N.								
₿	<b>æ</b>	ф	ф	Ф	⊕	ф	₿	
ф	ф	ф	ф	ф	ф	ф	ф	
⊉	Ф	Ф	Þ	ආ	ආ	Ф	Ф	
ദ	ഭാ	Ф	Ф	ക	ദ	Ф	ക	
æ	æ	æ	æ	ф	æ	a¢a	æ	
-5-	ക	ക	\$	ф	\$	ട	ත	
ക	ഷം	ф	₿	ക	ദ	ക	⊜	
Ф	⊅	Ф	₽	⊅	₽	⊅	⊅	
ഷ്ട	<b>B</b>	ቆ⊳	ക	ഭ	<b>B</b>	<b>B</b> P	ക	
ලා	ക	<b>\$</b> 0	<b>.</b>	අ	⊕	⊕	എം	

NB	AKIA				
		1	n	m	N
Ф	ආ	ው	ц Ср	Ð	сфр
¢	ф	ф	ф	ф	æ
2	മ	Ф	⊉	Ф	മ
З	Ф	æ	<b>3</b> 0	Ф	3
<b>4</b> Þ	<b>¢</b>	æ	a‡⊃	æ	a‡p
5	ஷ	ත	\$	ф	•
<b>6</b> 0	⊕	¢	₫	Ф	Ф
⊅	₽	Ф	Ф	æ	⊅
8	<b>B</b>	ക	ക	đ	₿
90	ф	œ	<b>G</b> D	c <b>9</b> 6	⊕
	_				

NUMERO / ZE

đ œ â

ф ф 1

æ 2 2

\$ ഭ 3

a‡⊃

ക ക c.

¢ ദ de ф

P Þ

æ ക 3

a: **\_** 9

4

4

#### MULTIPLE CHOICE QUESTIONS (Time: 45 minutes)

- 1. FREE-QUESTION. The capital of Spain is:
  - (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin
- 2. It is believed that a Poisson distribution of parameter equal to 9 can appropriately model the behavior of the variable that represents:
  - (A) the waiting time for clients in a queue
  - (B) the number of daily complaints that are handled in a given customer service office
  - (C) the height of a child in centimeters
  - (D) the number of clients, out of 9, that buy a given product
  - (E) the proportion of heads obtained in 9 throws of a coin

### Questions 3 to 5 refer to the following exercise:

A ticket vending machine for a given public transportation system is not working properly and it fails with probability 0.8. We assume independence between the different clients using the vending machine.

3. If 15 clients use the ticket vending machine, the probability that the machine fails for at least 12 of those clients is:

 $(A) 0.3980 (B) 0.6020 (C) 0.2502 (D) 0.7498 (E) 0.6482 \\ (E) 0.6482 (E) 0.648 (E) 0.648 (E) 0.648 (E) 0.$ 

4. If 60 clients use the ticket vending machine, the approximate probability that the machine fails for at most 49 of those clients is:

(A) 0.6844 (B) 0.6255 (C) 0.3156 (D) 0.3745 (E) 0.5158 (E

- 5. If 361 clients use the ticket vending machine, the approximate probability that the machine fails for exactly 291 of those clients is:
  - (A) 0.1400 (B) 0.6103 (C) 0.0535 (D) 0.6406 (E) 0.8849

#### Questions 6 to 8 refer to the following exercise:

It is believed that in a given firm the yearly number of employees' maternity leaves follows a Poisson distribution with mean equal to 4. We assume independence between the different employees' maternity leaves.

6. The probability that in a given year the firm has fewer than 6 employees' maternity leaves is:

$$(A) 0.11 (B) 0.05 (C) 0.89 (D) 0.21 (E) 0.79$$

7. The probability that in a 50-year period the firm has more than 180 employees' maternity leaves is:

(A) 0.08 (B) 0.51 (C) 0.95 (D) 0.05 (E) 0.92

8. The number k of employees' maternity leaves that satisfies P(X = k) = P(X = k - 1) is equal to: (A) 1 (B) 3 (C) 5 (D) 4 (E) 2

- 9. Let X and Y be two independent r.v. having exponential distributions with parameters  $\lambda_1$  and  $\lambda_2$ , respectively. The distribution of the r.v. X + Y is:
  - (A) exponential distribution with mean  $\lambda_1 + \lambda_2$ .
  - (B) exponential distribution with parameter  $\lambda$ , where  $\lambda = \lambda_2 = \lambda_1$
  - (C) exponential distribution with mean  $\frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ .
  - (D) exponential distribution with parameter  $\lambda_1 + \lambda_2$
  - (E) All false

#### Questions 10 and 11 refer to the following exercise:

Let X, Y and Z be three independent r.v. having, respectively, the following distributions:  $N(2, \sigma_X^2 = 4)$ ,  $N(3, \sigma_Y^2 = 9)$  and  $N(1, \sigma_Z^2 = 4)$ .

- 10. Among the following random variables, which one follows a  $\chi^2_{3|}$  distribution?
  - (A)  $\left(\frac{X-2}{2}\right)^2 + \left(\frac{Y-3}{3}\right)^2 + \left(\frac{Z-1}{2}\right)^2$ (B)  $\frac{X^2-2}{2} + \frac{Y^2-3}{3} + \frac{Z^2-1}{2}$ (C)  $\frac{X^2-2}{4} + \frac{Y^2-3}{9} + \frac{Z^2-1}{4}$ (D)  $X^2 + Y^2 + Z^2$
  - (E) All false
- 11. Among the following random variables, which one follows a  $t_{\overline{2}|}$  distribution?

(A) 
$$\frac{\frac{X-2}{2}}{\sqrt{\frac{\left(\frac{Y-3}{3}\right)^2 + \left(\frac{Z-1}{2}\right)^2}{2}}}$$
 (B)  $\frac{\frac{X-2}{2}}{\left(\frac{Y-3}{3}\right)^2 + \left(\frac{Z-1}{2}\right)^2}$  (C)  $\frac{X}{\sqrt{\frac{Y^2+Z^2}{2}}}$  (D)  $\frac{\frac{X-2}{4}}{\sqrt{\frac{Y^2+Z^2}{2}}}$  (E) All false

- 12. Let  $X_1, X_2, \ldots, X_{10}$  be independent and identically distributed r.v. having a  $N(0, \sigma^2)$  distribution,  $\sigma^2 > 0$ . The distribution of the r.v.  $Z = \frac{X_1^2 + \cdots + X_{10}^2}{\sigma^2}$  is:
  - (A) Unknown with mean 0 and variance 20
  - (B) Student's t with 10 degrees of freedom
  - (C)  $\chi^2$  with 10 degrees of freedom
  - (D) Normal
  - (E) All false
- 13. Let X be a r.v. having a  $\gamma(\frac{1}{2}, 5)$  distribution. If we have that  $P(a \le X \le b) = 0.85$  and  $P(X \le a) = 0.1$ , what would be the corresponding values of a and b so that these conditions hold?
  - (A) 0.0158 and 3.84
  - (B) 4.87 and 18.3
  - (C) 3.25 and 23.2
  - (D) 1.61 and 11.1
  - (E) All false

- 14. Let X be a r.v. having a  $\gamma(\frac{1}{2}, \frac{3}{2})$  distribution and Y another r.v., independent from X, having a N(0, 1) distribution. The distribution of the r.v.  $Z = X + Y^2$  is:
  - (A) Snedecor's F with (1, 2) degrees of freedom
  - (B)  $\chi^2$  with 2 degrees of freedom
  - (C)  $\chi^2$  with 4 degrees of freedom
  - (D) Student's t with 2 degrees of freedom
  - (E) All false

15. The characteristic function for the r.v. X is given by  $\psi_X(u) = \left(1 - \frac{iu}{6}\right)^{-3}$ . We then have that:

- (A) The variance of X is equal to 9
- (B) The mean of X is equal to  $\frac{1}{2}$
- (C) The variance of X is equal to  $\frac{1}{2}$
- (D) The mean of X is equal to 6
- (E) All false

# SOLUTIONS

1: C	6: E	11: A
2: B	7: E	12: C
3: E	8: D	13: B
4: A	9: E	14: C
5: C	10: A	15: B