INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has seven numbered sheets, going from 0.1 to 0.7. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as is illustrated in the example.
6. The maximum final grade for each of the parts of the exam (i.e., the multiple choice questions part and the exercises part) is 30 points. You will need to obtain 15 points in each part of the exam to pass it. However, exams having a multiple choice question part with grades greater than or equal to 14 could, under special circumstances, be compensated with a good grade in the exercises part of the exam.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises. In "Resit" (column II) you will write the number of times you have registered for an exam in this course *not including this one.*

Example:

$\underbrace{12545}$ PEREZ, Ernesto Exam type $\underbrace{0}$ Resit

MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 30 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) Londres (E) Pekin

2. Let $\{X_n\}_{n\in\mathcal{N}}$ be a sequence of random variables with characteristic function given by:

$$\psi_n(u) = \left(1 - \frac{iu}{n^3}\right)^{-1}$$

   The sequence of random variables will converge:

   (A) Only in distribution to $X = 0$

   (B) Only in distribution to $X = 1$

   (C) In distribution and probability to $X = 1$

   (D) In distribution and probability to $X = 0$

   (E) All false

3. Let $\{X_n\}_{n\in\mathcal{N}}$ be a sequence of random variables with probability mass function given by:

$$P_n(x) = \begin{cases} \frac{1}{2} & \text{if } x = -\frac{1}{n} \\ \frac{1}{2} & \text{if } x = \frac{1}{n} \end{cases}$$

   The sequence of random variables will converge:

   (A) In distribution, probability and quadratic mean to $X = \frac{1}{2}$

   (B) In distribution, probability and quadratic mean to $X = 0$

   (C) Only in distribution to $X = 0$

   (D) Only in distribution to $X = \frac{1}{2}$

   (E) All false

Questions 4 to 6 refer to the following exercise:

The probability that a client entering an appliance store buys a product is 0.20. We assume independence between the different clients entering the store.

4. If 20 clients enter the store, the probability that five of them buy a product is:

   (A) 0.8042 (B) 0.2182 (C) 0.1091 (D) 0.6296 (E) 0.1746

5. The probability that, among the 20 clients, at least ten of them buy a product is:

   (A) 0.0074 (B) 0.0026 (C) 0.9974 (D) 0.0006 (E) 0.9994

6. If there are now 225 clients entering the store, the approximate probability that no more than 41 clients buy a product is:

   (A) 0.3546 (B) 0.7190 (C) 0.2810 (D) 0.5398 (E) 0.4602

Questions 7 and 8 refer to the following exercise:

The number of cars **per minute** driving by a specific street crossing follows a Poisson distribution with mean 6. We assume independence between the different cars driving by this street crossing.

7. The probability that in a given minute exactly six cars drive by this street crossing is:

   (A) 0.6063 (B) 0.7440 (C) 0.4457 (D) 0.1606 (E) 0.1377

8. The probability that in **6 minutes** exactly 42 cars drive by this street crossing is, approximately:

   (A) 0.8599 (B) 0.0387 (C) 0.1788 (D) 0.1401 (E) 0.8212

Questions 9 to 11 refer to the following exercise:

Let X_1, X_2 and X_3 be independent normally distributed random variables having respective means -1, 0 and 1 and variances 9, 1 and 4.

9. The probability that the random variable $W = \dfrac{(X_1+1)^2}{9} + X_2^2$ takes on values smaller than 0.103 is:

   (A) 0.95 (B) 0.10 (C) 0.05 (D) 0.90 (E) 0.25

10. The probability that the random variable $Y = \dfrac{2X_2}{(X_3-1)}$ takes on values smaller than 1.376 is:

    (A) 0.40 (B) 0.60 (C) 0.10 (D) 0.80 (E) 0.20

11. The probability that the random variable Y^2 takes on values larger than 161 is:

    (A) 0.05 (B) 0.10 (C) 0.01 (D) 0.95 (E) 0.90

Questions 12 and 13 refer to the following exercise:

Let X and Y be independent random variables such that $X \in \gamma(a, r)$ and $Y \in \gamma(b, s)$.

12. If we define the random variable $W = 2aX + 2bY$, then W is distributed as:

    (A) $\gamma(2, r+s)$ (B) $\gamma(1/2, 2(r+s))$ (C) $\gamma(1/2, r+s)$ (D) $\exp(r+s)$ (E) $\gamma(2, 2(r+s))$

13. If we define the random variable $Z = (X/b) + (Y/a)$, then Z is distributed as:

    (A) $\gamma(ab, r+s)$ (B) $\gamma(a/b, r+s)$ (C) $\gamma(1/ab, r+s)$ (D) $\gamma(b/a, r+s)$ (E) χ^2_{r+s}

14. Let X be a random variable with probability density function given by:

    $$f(x, \theta) = \frac{1}{2^\theta \Gamma(\theta)}\, x^{\theta-1}\, e^{-x/2}, \; x > 0, \; \theta > 0$$

    In order to estimate the parameter θ, a random sample of size n, $X_1, \ldots, X_n$ has been taken. A sufficient statistic for the parameter θ is:

    (A) $\sum_{i=1}^n X_i$ (B) $\prod_{i=1}^n X_i$ (C) $\prod_{i=1}^n \ln X_i$ (D) $\prod_{i=1}^n \left(\frac{1}{X_i}\right)$ (E) $\sum_{i=1}^n X_i^2$

Questions 15 and 16 refer to the following exercise:

Let X be a random variable with probability mass function given by:

$$P(X=0)=\frac{1}{\theta};\quad P(X=1)=\frac{5}{2\theta}$$

$$P(X=2)=\frac{(\theta-4)}{\theta};\quad P(X=3)=\frac{1}{2\theta}$$

In order to estimate the parameter θ, a random sample of size $n=20$ has been taken and has provided the following results:

X	0	1	2	3
frequency	8	5	3	4

15. The maximum likelihood estimate of θ is:

(A) $\frac{40}{17}$ (B) $\frac{17}{80}$ (C) $\frac{80}{17}$ (D) $\frac{17}{40}$ (E) $\frac{60}{17}$

16. The method of moments estimate of θ is:

(A) $\frac{80}{17}$ (B) $\frac{17}{80}$ (C) $\frac{60}{17}$ (D) $\frac{17}{40}$ (E) $\frac{40}{17}$

17. Let X be a random variable with probability density function given by:

$$f(x;\theta)=\theta\, e^{-\theta x},\qquad x>0,\qquad \theta>0$$

In order to estimate the parameter θ, a random sample of size n: $X_1,\ldots,X_n$ has been taken. The maximum likelihood estimator of θ is:

(A) $\dfrac{1}{\overline{X}-1}$ (B) $1-\overline{X}$ (C) $\overline{X}$ (D) $\overline{X}-1$ (E) $\dfrac{1}{\overline{X}}$

Questions 18 to 20 refer to the following exercise:

Let Z be a random variable such that $Z\in b(p,n)$. In order to estimate the parameter p, the following two estimators are proposed:

$$T_1=\frac{Z}{n};\qquad T_2=\frac{Z+1}{n+2}$$

18. With regard to these two estimators, we can state that:

(A) Both estimators are unbiased (B) Only T_2 is unbiased (C) -

(D) Both estimators are biased (E) Only T_1 is unbiased

19. With regard to these two estimators, we can state that:

(A) Only T_1 is consistent (B) Only T_2 is consistent (C) Both estimators are consistent

(D) - (E) -

20. The mean square error (MSE) for the estimator T_1, for $p=1/2$, is:

(A) $\frac{1}{4}$ (B) $\frac{1}{4n}$ (C) $\frac{1}{2}$ (D) $\frac{4}{n}$ (E) $\frac{1}{2n}$

21. Let X be a random variable with probability density function given by:

$$f(x,\theta) = \frac{1}{\theta}\, e^{-\frac{x}{\theta}} \qquad x > 0, \qquad \theta > 0$$

In order to test the null hypothesis $\theta = \frac{1}{2}$ against the alternative hypothesis $\theta = 1$, a random sample of size $n = 1$ has been taken. For a given significance level, the most powerful critical region for this test is of the form:

(A) $X \in (C_1, C_2)$ (B) $X \in (C_1, C_2)^c$ (C) $X \leq C$ (D) $X \geq C$ (E) All false

Questions 22 and 23 refer to the following exercise:

Let X be a random variable with probability mass function given by:

$$P(X=-1) = 3\theta^3 \qquad P(X=0) = 1 - 6\theta^3 \qquad P(X=1) = 3\theta^3$$

In order to test the null hypothesis $\theta = 0.50$ against the alternative hypothesis $\theta = 0.20$, a random sample of size $n = 1$ has been taken and we decide to reject the null hypothesis if $X = 0$.

22. The significance level for this test is:

(A) 0.375 (B) 0.024 (C) 0.625 (D) 0.250 (E) 0.750

23. The probability of type II error for this test is:

(A) 0.048 (B) 0.952 (C) 0.375 (D) 0.250 (E) 0.750

Questions 24 and 25 refer to the following exercise:

The number of clients entering a specific bank branch per hour follows a Poisson distribution. The branch director considers the idea of opening a new bank counter for the public if the average number of clients entering the branch is at least 7. In order to test the hypothesis about the need to open a new bank counter in this branch, that is $H_0 : \lambda \geq 7$, against the alternative hypothesis $H_1 : \lambda < 7$, the director has information about the clients entering the branch for a given hour.

24. At the $\alpha = 0.10$ significance level, the most powerful decision rule will be to reject H_0 if the number of clients entering this specific bank branch per hour is:

(A) $X \geq 3$ (B) $X \geq 4$ (C) $X \leq 7$ (D) $X \leq 4$ (E) $X \leq 3$

25. For this test and for a value of $\lambda = 5$, the probability of type II error will be:

(A) 0.8753 (B) 0.2650 (C) 0.1247 (D) 0.7350 (E) 0.1404

Questions 26 and 27 refer to the following exercise:

An individual is interested in buying a specific type of DVD reader. Before doing so, the individual asks for the price of this reader in 31 stores, obtaining a sample average price of 105 euros with a sample standard deviation of 20 euros. We assume normality.

26. With 90% confidence, we can state that the mean price of this specific DVD reader will be in the interval:

(A) (97.55, 112.45) (B) (100.22, 109.78) (C) (99.25, 110.75)
(D) (98.79, 111.21) (E) (101.35, 108.65)

27. With 90% confidence, we can state that the variance of this specific DVD reader will be in the interval:

(A) (307.69, 601.94) (B) (14.16, 33.51) (C) (283.11, 670.27)
(D) (255.64, 706.34) (E) (326.44, 585.45)

Questions 28 and 29 refer to the following exercise:

A researcher from the School of Biology at the University of the Basque Country wishes to estimate the proportion of rats that, having been exposed to a certain risk factor, develop lung cancer. The researcher randomly selects 150 rats from the ones about which it was known both that they have been exposed to this risk factor during the last few years and their status on the development or not of any form of lung cancer. The collected data indicated that, among the 150 rats, 57 developed lung cancer.

28. The 95% confidence interval for the proportion of rats that, having been exposed to the risk factor, will eventually develop lung cancer is, approximately:

(A) (0.34, 0.42) (B) (0.32, 0.44) (C) (0.25, 0.51) (D) (0.28, 0.48) (E) (0.30, 0.46)

29. At the 5% significance level, we wish to test the null hypothesis that the proportion of rats exposed to the risk factor that will develop lung cancer is **greater than or equal** to 0.40. The decision will be:

(A) Do not reject the null hypothesis (B) It cannot be decided (C) Reject the null hypothesis
(D) - (E) -

30. We wish to test if the type of health care a patient has in an emergency is related or not to the area of residence. In order to do it, a random sample has been taken and patients are then classified according to their area of residence (urban area or rural area) and the type of health care they received in case of an emergency (domestic health care, closer state-health service hospital, or closer large hospital). The test that one has to carry out is:

   (A) Homogeneity
   (B) Independence
   (C) Means comparison
   (D) Goodness of fit to a totally specified distribution
   (E) Goodness of fit to a partially specified distribution

EXERCISES (Time: 75 minutes)

A. (10 points, 25 minutes)

Let X be a continuous random variable with probability density function given by:

$$f(x,\theta) = \begin{cases} \dfrac{2\theta}{(1-\theta)}\, x^{(3\theta-1)/(1-\theta)} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, \ldots, X_n$ has been taken. **Hint**: It is known that $\mathrm{E}(X) = \frac{2\theta}{\theta+1}$.

a) Find, **providing all relevant details**, the method of moments estimator of θ.

b) Find, **providing all relevant details**, the maximum likelihood estimator of θ.

B. (10 points, 25 minutes)

The following table describes the probability mass function for the discrete random variable X under the null ($P_0(x)$) and alternative ($P_1(x)$) hypotheses.

X	1	2	3	4	5	6
$P_0(x)$	0	0	0.05	0.10	0.40	0.45
$P_1(x)$	0.35	0.30	0.15	0.20	0	0

In order to test the null hypothesis $H_0 : P(x) = P_0(x)$ against the alternative hypothesis $H_1 : P(x) = P_1(x)$, a random sample of size $n = 1$ has been taken.

a) Would you include the points $X = \{1, 2\}$ in the critical region for this test? Explain.

b) Would you include the points $X = \{5, 6\}$ in the critical region for this test? Explain.

c) At the $\alpha = 10\%$ significance level, and providing all relevant details, find the most powerful critical region for this test. **Remark**: Before answering this item, recall the answers provided in the two items above.

C. (10 points, 25 minutes)

A firm wants to sell three types of MP3 devices and, regarding this, it has the following information:

Type	1	2	3
Probabilities	$(1-4\theta)$	2θ	2θ

a) In order to estimate these probabilities, a random sample of 50 individuals has been taken and has provided the following information: 20 individuals bought type 1 MP3 devices; 20 individuals bought type 2 MP3 devices and 10 individuals bought type 3 MP3 devices. Find the maximum likelihood estimate of θ.

b) At the $\alpha = 5\%$ significance level, test the hypothesis that the probability distribution the firm has is the correct one.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS

1: C	11: A	21: D
2: D	12: C	22: D
3: B	13: A	23: A
4: E	14: B	24: E
5: B	15: C	25: D
6: C	16: A	26: D
7: D	17: E	27: C
8: B	18: E	28: E
9: C	19: C	29: A
10: D	20: B	30: B

SOLUTIONS TO EXERCISES

A) The probability density function for the random variable X is given by

$$f(x,\theta) = \begin{cases} \dfrac{2\theta}{(1-\theta)}\; x^{(3\theta-1)/(1-\theta)} & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, \ldots, X_n$, has been taken. Moreover, it is known that $\mathrm{E}(X) = \frac{2\theta}{\theta+1}$.

a) Method of moments estimator

In order to find the method of moments estimator of θ, we equate the first population moment to the first sample moment. That is,

$$\alpha_1 = \mathrm{E}(X) = a_1 = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X}$$

Given that it is known that $\mathrm{E}(X) = \frac{2\theta}{\theta+1}$, we have that:

$$\alpha_1 = \mathrm{E}(X) = \frac{2\theta}{\theta+1} = a_1 = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X} \Longrightarrow 2\theta = \overline{X}\theta + \overline{X}$$

$$\Longrightarrow \theta(2-\overline{X}) = \overline{X} \Longrightarrow \hat{\theta}_{MM} = \frac{\overline{X}}{(2-\overline{X})}$$

b) Maximum likelihood estimator

The likelihood function for the sample is given by:

$$\begin{aligned} L(\theta) &= f(x_1;\theta)\ \ldots\ f(x_n;\theta) \\ &= \left[\frac{2\theta}{(1-\theta)}\; x_1^{(3\theta-1)/(1-\theta)}\right] \cdots \left[\frac{2\theta}{(1-\theta)}\; x_n^{(3\theta-1)/(1-\theta)}\right] \\ &= \frac{2^n\theta^n}{(1-\theta)^n}\; \left[\prod_{i=1}^{n} x_i\right]^{(3\theta-1)/(1-\theta)} \end{aligned}$$

We now compute its natural logarithm to obtain:

$$\ln L(\theta) = n\ln 2 + n\ln\theta - n\ln(1-\theta) + \left(\frac{3\theta-1}{1-\theta}\right)\; \ln\left[\prod_{i=1}^{n} x_i\right]$$

If we take derivatives with respect to θ and equate this to zero, we will have that:

$$\frac{\partial \ln L(\theta)}{\partial\theta} = \frac{n}{\theta} + \frac{n}{(1-\theta)} + \ln\left[\prod_{i=1}^{n} x_i\right] \left[\frac{(1-\theta)(3) - (3\theta-1)(-1)}{(1-\theta)^2}\right] = 0$$

$$\Longrightarrow \frac{n}{\theta} + \frac{n}{(1-\theta)} + \left[\frac{2}{(1-\theta)^2}\right] \ln\left[\prod_{i=1}^{n} x_i\right] = 0,$$

$$\Longrightarrow \left[\frac{2}{(1-\theta)^2}\right] \ln\left[\prod_{i=1}^{n} x_i\right] = -\frac{n}{\theta} - \frac{n}{(1-\theta)} = -\left[\frac{n(1-\theta)+n\theta}{\theta(1-\theta)}\right] = -\frac{n}{\theta(1-\theta)}$$

$$\Longrightarrow \left[\frac{2}{(1-\theta)}\right] \ln\left[\prod_{i=1}^{n} x_i\right] = -\frac{n}{\theta}$$

$$\Longrightarrow 2\theta \ln\left[\prod_{i=1}^{n} x_i\right] = -n + n\theta \Longrightarrow n = \theta\left\{n - 2\ln\left[\prod_{i=1}^{n} x_i\right]\right\}$$

so that,

$$\hat{\theta}_{MV} = \frac{n}{\{n - 2\ln\left[\prod_{i=1}^{n} X_i\right]\}} = \frac{n}{[n - 2\sum_{i=1}^{n} \ln(X_i)]}$$

B) We wish to test the null hypothesis that X is a discrete random variable with probability mass function $P_0(x)$ against the alternative hypothesis that its probability mass function is $P_1(x)$:

X	1	2	3	4	5	6
$P_0(x)$	0	0	0.05	0.10	0.40	0.45
$P_1(x)$	0.35	0.30	0.15	0.20	0	0

A random sample of size $n = 1$ has been taken; that is, we observe X.

a) Would you include the points $X = \{1, 2\}$ in the critical region for this test?

Given that, under the probability mass function in the null hypothesis $P_0(x)$, these points have zero probability, the random variable X cannot take on these values under the null hypothesis. Therefore, $X = \{1, 2\}$ are rejection points for H_0 and, thus, **should always** be included in the critical region for this test.

b) Would you include the points $X = \{5, 6\}$ in the critical region for this test?

Given that, under the probability mass function in the alternative hypothesis $P_1(x)$, these points have zero probability, the random variable X cannot take on these values under the alternative hypothesis, but it can clearly take on these values under the null hypothesis. Therefore, $X = \{5, 6\}$ are not rejection points for H_0 and, thus, **should never** be included in the critical region for this test.

c) At the $\alpha = 0.10$ significance level and recalling the answers provided in earlier items for this exercise, we have that there are only two possible critical regions for this test: $\text{CR}_1 = \{1, 2, 3\}$ and $\text{CR}_2 = \{1, 2, 4\}$. This is due to the fact that:

$$\alpha_1 = P(X \in \text{CR}_1 | P_0) = P(X = 1, 2, 3 | P_0) = 0 + 0 + 0.05 = 0.05 \leq \alpha = 0.10$$
$$\alpha_2 = P(X \in \text{CR}_2 | P_0) = P(X = 1, 2, 4 | P_0) = 0 + 0 + 0.10 = 0.10 \leq \alpha = 0.10$$

In order to see which one of these two critical regions is the most powerful one, we compute their respective powers:

$$\text{Power}_1 = P(X \in \text{CR}_1 | P_1) = P(X = 1, 2, 3 | P_1) = 0.35 + 0.30 + 0.15 = 0.80$$
$$\text{Power}_2 = P(X \in \text{CR}_2 | P_1) = P(X = 1, 2, 4 | P_1) = 0.35 + 0.30 + 0.20 = 0.85$$

From these calculations, we conclude that, at the $\alpha = 0.10$ significance level, the most powerful critical region for this test is CR_2.

C) It is a **goodness of fit test to a partially specified distribution**. The information we have to perform this test is given below:

Type	1	2	3
Probabilities	$(1-4\theta)$	2θ	2θ

a) In order to be able to estimate these probabilities, a random sample of 50 individuals has been taken and has provided the following information: 20 individuals bought type 1 MP3 devices; 20 individuals bought type 2 MP3 devices and 10 individuals bought type 3 MP3 devices. With this information and to find the maximum likelihood estimator of θ, we have to write the likelihood function:

$$L(\theta) = (1-4\theta)^{20}(2\theta)^{20}(2\theta)^{10} = 2^{30}\theta^{30}(1-4\theta)^{20}$$

We now compute its natural logarithm to obtain:

$$\ln L(\theta) = 30\ln 2 + 30\ln(\theta) + 20\ln(1-4\theta)$$

If we take derivatives with respect to θ and equate this to zero, we will have that:

$$\frac{\partial \ln L(\theta)}{\partial\theta} = \frac{30}{\theta} - \frac{80}{(1-4\theta)} = 0 \Longrightarrow 30(1-4\theta) = 80\theta \Longrightarrow 30 - 120\theta = 80\theta$$

$$\Longrightarrow 30 = 200\theta \Longrightarrow \hat{\theta}_{MV} = \frac{30}{200} = 0.15$$

b) Goodness of fit test to a partially specified distribution.

First of all, we have that the estimated probabilities, p_i, for each of the MP3 devices will be:

$$P(\text{Type } 1) = (1-4(0.15)) = 0.40; \qquad P(\text{Type } 2) = P(\text{Type } 3) = 2(0.15) = 0.30$$

Given that we have estimated the parameter θ, we have that $h = 1$. Moreover, as we have three types of MP3 devices, $k = 3$. With this information and in order to perform the test, we build the following table:

	n_i	$\hat{p}_i$	$n\hat{p}_i$	$\frac{(n_i - n\hat{p}_i)^2}{n\hat{p}_i}$
Type 1	20	0.40	20	0
Type 2	20	0.30	15	1.67
Type 3	10	0.30	15	1.67
	$n = 50$	1	$n = 50$	$z \simeq 3.34$

Under the null hypothesis that the probability distribution the firm has is the correct one, the test statistic $\sum_i \frac{(n_i - n\hat{p}_i)^2}{n\hat{p}_i} \sim \chi^2_{k-h-1}$, where k is the number of types of MP3 devices ($k = 3$) and h is the number of estimated parameters ($h = 1$).

The decision rule, at a $\alpha = 5\%$ approximate significance level, will be to reject the null hypothesis if:

$$z > \chi^2_{k-h-1\,,\,0.05} = \chi^2_{1,0.05}$$

In this case:

$$z = 3.34 < 3.84 = \chi^2_{1,0.05}$$

so that, at the $\alpha = 5\%$ approximate significance level, we do not reject the null hypothesis that the probability distribution the firm has for the different types of MP3 devices is the correct one.