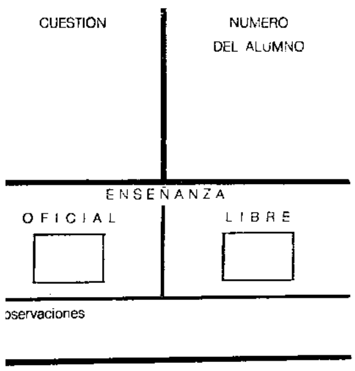
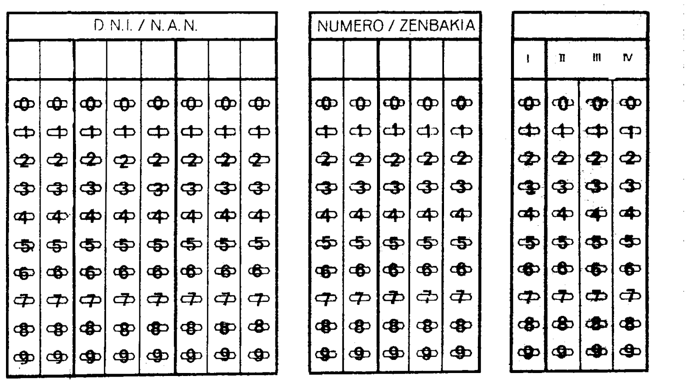
INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has six numbered sheets, going from 0.1 to 0.6. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as is illustrated in the example.
6. The maximum final grade for each of the parts of the exam (i.e., the multiple choice questions part and the exercises part) is 30 points. You will need to obtain 15 points in each part of the exam to pass it. However, exams having a multiple choice question part with grades greater than or equal to 14 could, under special circumstances, be compensated with a good grade in the exercises part of the exam.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises. In "Resit" (column II) you will write the number of times you have registered for an exam in this course *not including this one.*

Example:

$\underbrace{12545}$ PEREZ, Ernesto Exam type $\underbrace{0}$ Resit





MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 40 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) London (E) Pekin

Questions 2 to 4 refer to the following exercise:

The area (in square meters) of the apartments that are on sale in a given residential area follows a normal $N(90, \sigma = 10)$ distribution. Apartments having an area of at least 98.4 square meters are considered as large apartments. In addition, we assume independence between the areas for the different apartments on sale. **Remark**: In order to compute this probability you should round it off up to one decimal place.

2. If a random sample of 10 apartments on sale is taken in that area, the probability that at least 6 of them are large is:

   (A) 0.9991 (B) 0.9936 (C) 0.0064 (D) 0.0055 (E) 0.0009

3. In the same random sample of 10 apartments, the probability that exactly 8 of them **are not** large is:

   (A) 1 (B) 0.2013 (C) 0.0001 (D) 0.3020 (E) 0.6778

4. If we now take a random sample of 150 apartments, the probability that at least 35 of them are large is:

   (A) 0.2061 (B) 0.4286 (C) 0.8212 (D) 0.5714 (E) 0.1788

Questions 5 to 7 refer to the following exercise:

The number of students that arrive per minute at a given University Administrative Office follows a Poisson distribution with mean equal to 1. We assume independence between students' arrivals in different minutes.

5. The probability that in one minute fewer than 2 students go to the University Administrative Office is:

   (A) 0.3679 (B) 0.0803 (C) 0.1839 (D) 0.9197 (E) 0.7358

6. The probability that in a 5-minute period at least 6 students go to the University Administrative Office is:

   (A) 0.7622 (B) 0.2378 (C) 0.1334 (D) 0.6160 (E) 0.3840

7. The approximate probability that in a one-hour period exactly 60 students go to the University Administrative Office is:

   (A) 0.0956 (B) 0.0478 (C) 0.1478 (D) 0.5239 (E) 0.0239

8. Let $\{X_n\}_{n \in \mathcal{N}}$ be a sequence of random variables having a $N(1, \sigma^2 = 1/n^3)$ distribution. The sequence will converge:

   (A) In distribution, probability and quadratic mean to $X = 1$

   (B) Only in Probability to $X = 1$

   (C) Only in distribution to $X = 1$

   (D) Only in distributuon and probability to $X = 1$

   (E) All false

9. Let $\{X_n\}_{n\geq 2}$ be a sequence of random variables defined as follows:

$$P(X_n = -n^2) = \frac{1}{n^3}; \qquad P(X_n = 0) = 1 - \frac{2}{n^3}; \qquad P(X_n = n^2) = \frac{1}{n^3}$$

The sequence will converge:

(A) Only in probability to $X = 0$

(B) Only in distribution to $X = 0$

(C) Only in distribution and probability to $X = 0$

(D) In distribution, probability and quadratic mean to $X = 0$

(E) In distribution and quadratic mean to $X = 0$

10. Let X, Y and Z be independent r.v. such that $X \in \gamma(1/2, 1)$, $Y \in \gamma(1/2, 2)$ and $Z \in \gamma(2, 3)$. The distribution of the r.v. $X + Y + Z$ is:

(A) $\chi^2_{\overline{6}|}$ (B) $\gamma(1/2, 3)$ (C) $\gamma(2, 6)$ (D) All false (E) $\gamma(1/2, 6)$

11. Let X and Y be independent r.v. having a $N(0, 1)$ distribution. Let $Z = (X/Y)^2$. The distribution of the r.v. Z is:

(A) t_1 (B) $N(0, 1)$ (C) $\chi^2_{\overline{1}|}$ (D) $\mathcal{F}_{\overline{1,1}|}$ (E) $Z \equiv 1$

12. Let X be a random variable having a $t_{\overline{25}|}$ distribution. Then, $P(-2.48 < X < -0.684)$ is:

(A) 0.02 (B) 0.50 (C) 0.24 (D) 0.56 (E) 0.74

13. Let X be a random variable having an exponential distribution with media $\frac{1}{2}$. Then, the approximate value of $P(-1.5 < X < 1/4)$ is:

(A) 0.39 (B) 0.86 (C) 0.14 (D) 0.22 (E) 0.61

14. Let X be a random variable having an $\mathcal{F}_{5,8}$ distribution. Then, the value of k such that $P(X > k) = 0.95$ is:

(A) 0.21 (B) 3.69 (C) 0.27 (D) 3.34 (E) 4.82

Questions 15 and 16 refer to the following exercise:

In a shooting game a player throw darts, continuously and identically, over his/her objective. We wish to estimate the probability θ that a given dart hits the objective.

15. If the player needed 13 throws to hit the objective, the maximum likelihood estimate of θ is:

(A) $\frac{12}{13}$ (B) $\frac{1}{13}$ (C) $\frac{1}{12}$ (D) $\frac{13}{14}$ (E) $\frac{11}{12}$

16. After the previous game (i.e., the one requiring 13 throws of the dart to hit the objective), the player tries again one more time and needs 15 throws of the dart to hit the objective. The new maximum likelihood estimate of θ considering all dart throws is:

(A) $\frac{14}{15}$ (B) $\frac{1}{14}$ (C) $\frac{1}{28}$ (D) $\frac{1}{17}$ (E) $\frac{12}{13}$

Questions 17 to 20 refer to the following exercise:

Let X be a random variable having a uniform $U[\theta-1,\theta+1]$ distribution and that, in order to estimate the parameter θ, we have taken a r.s. of size n, $X_1,\ldots,X_n$, from it. We propose to use the estimator $\hat{\theta}=\overline{X}+\frac{1}{n^2}$. In addition, we know that the variance of a $U[a,b]$ r.v. is $\sigma^2=(b-a)^2/12$.

17. Is this estimator unbiased?

(A) No (B) - (C) Yes (D) - (E) -

18. The variance of this estimator is:

(A) $\frac{1}{3}$ (B) $\frac{1}{12n}$ (C) $\frac{1}{3n}$ (D) $\frac{1}{12n}+\frac{1}{n^2}$ (E) $\frac{1}{12}$

19. The mean square error of this estimator is:

(A) $\frac{1}{12n}$ (B) $\frac{1}{3n}+\frac{1}{n^4}$ (C) $\frac{1}{12}$ (D) $\frac{1}{12n}+\frac{1}{n^2}$ (E) $\frac{1}{3}+\frac{1}{n^4}$

20. Is this estimator consistent?

(A) Yes (B) - (C) - (D) - (E) No

Questions 21 to 23 refer to the following exercise:

Let X be a r.v. with probability density function given by

$$f(x;\theta)=\theta^2x^{\theta^2-1},\qquad 0<x\le 1,\qquad \theta>0$$

We wish to test the null hypothesis $H_0:\theta=1$ against the alternative hypothesis $H_1:\theta=2$. In order to do so, a random sample of size $n=1$ has been taken (that is, we observe X).

21. The most powerful critical region for X will be of the form:

(A) $(0,K)$ (B) (K_1,K_2) (C) $(K,1)$ (D) $(K_1,K_2)^{\mathrm{c}}$ (E) All false

22. If $\alpha=0.10$, the most powerful critical region will be:

(A) $(0.95,1)$ (B) $(0.10,0.90)$ (C) $(0.90,1)$ (D) $[0.10,0.90]^c$ (E) $(0,0.10)$

23. The power of this test is:

(A) 0.0975 (B) 0.3439 (C) 0.9025 (D) 0.6561 (E) 0.0025

Questions 24 and 25 refer to the following exercise:

Let $X_1,\ldots,X_n$ be a r.s. of size 20 taken from a binary $b(p)$ population. We wish to test the null hypothesis $H_0:p=0.20$ against the alternative hypothesis $H_1:p=0.40$. Let $Z=\sum_{i=1}^n X_i$, the number of total successes in the 20 repetitions of the binary experiment.

24. At the 5% significance level, the most powerful critical region is:

(A) $Z\ge 8$ (B) $Z\in[2,8]$ (C) $Z\ge 7$ (D) $Z\in[2,8]^{\mathrm{c}}$ (E) $Z\le 2$

25. The power of this test is:

(A) 0.4159 (B) 0.5956 (C) 0.5841 (D) 0.4044 (E) 0.2447

Questions 26 and 27 refer to the following exercise:

In a manufacturing process, we wish to test the null hypothesis that the percentage of acceptable manufactured parts is al least 10%, against the alternative that it is smaller than 10%. We have taken a random sample of size $n = 300$ observing 24 acceptable parts.

26. With an approximate 95% confidence, we can state that the proportion of acceptable manufactured parts produced falls in the interval:

(A) $(0.02,\ 0.14)$ (B) $(0,\ 0.15)$ (C) $(0.01,\ 0.15)$ (D) $(0,\ 0.08)$ (E) $(0.05,\ 0.11)$

27. At the approximate 5% significance level, the decision of the test will be:

(A) Do not reject the null hypothesis (B) - (C) Reject the null hypothesis (D) - (E) -

Questions 28 and 29 refer to the following exercise:

Let X and Y be two independent r.v. such that $X \in N(m_X, \sigma_X^2 = 25)$ and $Y \in N(m_Y, \sigma_Y^2 = 36)$. In order to test $H_0 : m_X = m_Y$ against $H_1 : m_X \neq m_Y$, we have taken two separate r.s. of size 30 each, so that $\overline{x} = 82$ and $\overline{y} = 80$.

28. A 90% confidence interval for $(m_{X-}m_Y)$ is, approximately:

(A) $(-0.34,\ 4.34)$ (B) $(-1.33,\ 5.33)$ (C) $(0.79,\ 4.79)$ (D) $(0.34,\ 4.34)$ (E) $(-0.79,\ 4.79)$

29. At the 10% significance level, the decision of this test will be:

(A) Do not reject H_0 (B) - (C) Reject H_0 (D) - (E) -

30. It is known that the expense in candies in a given population follows a normal distribution and it is claimed that the variance of these expenses made by individuals younger than 20 years, σ_1^2, is larger than or equal that the variance in these expenses made by older individuals, σ_2^2. In order to test $H_0 : \sigma_1^2 \geq \sigma_2^2$ against $H_1 : \sigma_1^2 < \sigma_2^2$, we have taken two separate r.s. of size 31 in each of the two populations, so that $s_1^2 = 62$ and $s_2^2 = 65$. At the 10% significance level, the decision of the test will be:

(A) Reject H_0 (B) - (C) - (D) - (E) Do not reject H_0

EXERCISES (Time: 75 minutes)

A (10 points, 25 minutes)

Let $X_1, \ldots, X_n$ be a r.s. of size n taken from a r.v. X having probability density function given by:

$$f(x, \theta) = \frac{1}{24\theta^5}\, x^4\, e^{-\frac{x}{\theta}} \qquad x > 0, \qquad \theta > 0$$

It is known that $\mathrm{E}(X) = 5\theta$ and $\mathrm{Var}(X) = 5\theta^2$.

i) Find (providing all relevant details) the maximum likelihood estimator of the parameter θ.

ii) Find (providing all relevant details) the method of moments estimator of the parameter θ.

iii) Is the method of moments estimator of θ unbiased?, consistent? and efficient?

B. (10 points, 25 minutes)

Let $X_1, \ldots, X_4$ be a r.s. taken from a population having a Poisson distribution with parameter λ. We wish to test the null hypothesis $H_0 : \lambda = 1$ against the alternative hypothesis $H_1 : \lambda = 2$.

i) Find (providing all relevant details) the form of the most powerful critical region for this test.

ii) At the 10% significance level, compute the critical region for this test.

iii) What is the power for this test?

C. (10 points, 25 minutes) In a given University, we wish to test if grades in a Statistics course depend on the field a given student is in. In order to do so, a random sample of students regularly attending class is taken, classifying them according to their final grade and their field of study, providing the following results:

	Psychology	Medicine	Pharmacy	Totals
Outstanding	11	28	22	61
Very good	20	34	30	84
Good	22	8	13	43
Fail	6	4	9	19
Totals	59	74	74	207

Use these data to perform the requested test at the 5% significance level.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS

1: C	11: D	21: C
2: C	12: C	22: C
3: D	13: A	23: B
4: E	14: A	24: A
5: E	15: B	25: C
6: E	16: B	26: E
7: B	17: A	27: A
8: A	18: C	28: A
9: C	19: B	29: A
10: D	20: A	30: E

SOLUTIONS TO EXERCISES

Exercise A

The probability density function for the r.v. X is:

$$f(x,\theta) = \frac{1}{24\theta^5} x^4 e^{-\frac{1}{\theta}x} \qquad x > 0, \qquad \theta > 0$$

We know that $\mathrm{E}(X) = 5\theta$ and $\mathrm{Var}(X) = 5\theta^2$.

i) Method of moments estimator

We make the first population moment equal to its corresponding sample moment. That is,

$$\alpha_1 = \mathrm{E}(X) = a_1 = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X}$$

From the r.v. X we know that $\mathrm{E}(X) = 5\theta$, so that we will have that:

$$5\theta = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X}$$

and, thus,

$$\hat{\theta}_{MM} = \frac{\overline{X}}{5}$$

ii) Maximum likelihood estimator

The likelihood function for the sample is:

$$\begin{aligned} L(\theta) &= f(x_1;\theta) \ \dots \ f(x_n;\theta) = \\ &= \left(\frac{1}{24\theta^5} x_1^4 e^{-\frac{x_1}{\theta}}\right) \ \dots \ \left(\frac{1}{24\theta^5} x_n^4 e^{-\frac{x_n}{\theta}}\right) = \\ &= \frac{1}{24^n \theta^{5n}} \left(\prod_{i=1}^{n} x_i\right)^4 e^{-\frac{\sum_{i=1}^{n} x_i}{\theta}} \end{aligned}$$

If we take its natural, we have that:

$$\ln L(\theta) = -n\ln(24) - 5n\ln\theta + \ln\left[\prod_{i=1}^{n} x_i\right]^4 - \frac{1}{\theta}\sum_{i=1}^{n} x_i$$

If we know take its derivative with respect to θ and make it equal to zero, we have:

$$\frac{\partial \ln L(\theta)}{\partial\theta} = \frac{-5n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} x_i = 0$$

so that,

$$-5n\theta + \sum_{i=1}^{n} x_i = 0$$

$$\hat{\theta}_{ML} = \frac{\sum_{i=1}^{n} X_i}{5n} = \frac{\overline{X}}{5}$$

iii) Unbiasedness:

In order to see if the method of moments estimator is unbiased we have to verify if $\mathrm{E}(\hat{\theta}_{MM}) = \theta$ holds.

In this case,

$$\mathrm{E}(\hat{\theta}_{MM}) = \mathrm{E}\left(\frac{1}{5}\ \overline{X}\right) = \frac{1}{5}\ \mathrm{E}(X) = \frac{1}{5}\ (5\theta) = \theta$$

Therefore, it is unbiased.

Consistency

In order to see if the method of moments estimator is consistent, we compute its variance.

$$\mathrm{Var}(\hat{\theta}_{MM}) = \mathrm{Var}\left(\frac{1}{5}\ \overline{X}\right) = \frac{1}{25}\ \frac{\mathrm{Var}(X)}{n} = \frac{1}{25}\ \frac{5\theta^2}{n} = \frac{\theta^2}{5n}$$

Given that it is an unbiased estimator and that its variance goes to zero n goes to infinity, the two sufficient conditions for its consistency hold and, therefore, it is a consistent estimator of θ.

Efficiency

In order to see if the method of moments estimator is efficient, we have to check if its variance coincides with the Cramer-Rao lower bound for a regular and unbiased estimator of θ.

The bound is $L_c = \dfrac{1}{n\mathrm{E}\left[\dfrac{\partial \ln(f(x,\theta))}{\partial\theta}\right]^2}$

To compute it we have that:

$$\ln f(x,\theta) = -\ln(24) - 5\ln(\theta) + 4\ln x - \frac{x}{\theta}$$

$$\frac{\partial \ln f(x,\theta)}{\partial\theta} = -\frac{5}{\theta} + \frac{x}{\theta^2} = \frac{1}{\theta^2}(x - 5\theta)$$

$$\left[\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right]^2 = \frac{1}{\theta^4}(x-5\theta)^2$$

$$\mathrm{E}\left[\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right]^2 = \frac{1}{\theta^4}\mathrm{E}(X-5\theta)^2 = \frac{1}{\theta^4}\mathrm{Var}(X) = \frac{5\theta^2}{\theta^4} = \frac{5}{\theta^2}$$

$$n\mathrm{E}\left[\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right]^2 = \frac{5n}{\theta^2}$$

y

$$L_c = \frac{\theta^2}{5n} = \mathrm{Var}(\hat{\theta}_{MV})$$

Thus, $\hat{\theta}_{MM}$ is an efficient estimator of θ.

Exercise B

Let $X_1, \ldots, X_n$ be a r.s. of size $n = 4$ taken from a Poisson distribution with parameter λ. We wish to test the null hypothesis $H_0 : \lambda = 1$ against the alternative hypothesis $H_1 : \lambda = 2$.

i) In order to obtain the form of the most powerful critical region we use Neyman-Pearson's Theorem. In this way, the corresponding likelihood functions under the null and alternative hypotheses will be given by:

$$L(\vec{x}; \lambda_0) = L(\vec{x}; \lambda = 1) = \left(e^{-n} (1)^{\sum_{i=1}^n x_i}\right) / \prod_{i=1}^n x_i!,$$

and

$$L(\vec{x}; \lambda_1) = L(\vec{x}; \lambda = 2) = \left(e^{-2n} (2)^{\sum_{i=1}^n x_i}\right) / \prod_{i=1}^n x_i!,$$

respectively. Therefore, if we use Neyman-Pearson's Theorem, we will have that:

$$\frac{L(\vec{x}; \lambda_0)}{L(\vec{x}; \lambda_1)} = \frac{\left(e^{-n} (1)^{\sum_{i=1}^n x_i}\right) / \prod_{i=1}^n x_i!}{\left(e^{-2n} (2)^{\sum_{i=1}^n x_i}\right) / \prod_{i=1}^n x_i!} \leq K, \;\; K > 0$$

$$\Longrightarrow e^n \left(\frac{1}{2}\right)^{\sum_{i=1}^n x_i} \leq K \Longrightarrow \left(\frac{1}{2}\right)^{\sum_{i=1}^n x_i} \leq K_1$$

If we take natural logarithms, we have that:

$$\left(\sum_{i=1}^n x_i\right) \ln\left(\frac{1}{2}\right) \leq K_2, \;\; K_2 > 0$$

Now, as $0.50 < 1$, the natural logarithm is negative, so that the inequality sign changes and, thus, we will have that the decision rule will be to reject the null hypothesis if $Z = \sum_{i=1}^n X_i \geq C$.

ii) At the $\alpha = 0.10$ significance level, and by taking into account that $Z = \sum_{i=1}^n X_i \in \mathcal{P}(n\lambda)$ and that $n = 4$, we will have that:

$$\alpha = 0.10 \geq P[Z \geq C | H_0] = P[Z \geq C | Z \in \mathcal{P}(4)] = 1 - F_Z(C - 1)$$

$$\Longrightarrow F_Z(C - 1) \geq 0.90 \Longrightarrow (C - 1) = 7 \Longrightarrow C = 8.$$

That is, we reject the null hypothesis if $Z = \sum_{i=1}^n X_i \geq 8$.

iii) To compute the power of the test, we will have that:

$$\text{Power} = P[Z \geq 8 | H_1] = P[Z \geq 8 | Z \in \mathcal{P}(8)] = 1 - F_Z(7) = 1 - 0.452961 = 0.547039.$$

Exercise C

We have to carry out a **test of independence**. The available data are summarized as follows:

	Psychology (PSY)	Medicine (MED)	Pharmacy (PHA)	Totals
Outstanding (OUTS)	11	28	22	61
Very good (VG)	20	34	30	84
Good (G)	22	8	13	43
Fail (F)	6	4	9	19
Totals	59	74	74	207

The corresponding probabilities $\hat{p}_{i\cdot}$ and $\hat{p}_{\cdot j}$ are estimated from the data. In this way,

$$\hat{p}_{\mathrm{OUTS},\bullet} = \frac{61}{207} \qquad \hat{p}_{\mathrm{VG},\bullet} = \frac{84}{207} \qquad \hat{p}_{\mathrm{G},\bullet} = \frac{43}{207} \qquad \hat{p}_{\mathrm{F},\bullet} = \frac{19}{207}$$

$$\hat{p}_{\bullet,\mathrm{PSY}} = \frac{59}{207} \qquad \hat{p}_{\bullet,\mathrm{MED}} = \frac{74}{207} \qquad \hat{p}_{\bullet,\mathrm{PHA}} = \frac{74}{207}$$

We then build the table that will allow us to obtain the required information to carry out the required test:

	n_{ij}	$\hat{p}_{i,j} = \hat{p}_{i,\bullet} \cdot \hat{p}_{\bullet,j}$	$n\hat{p}_{i,j}$	$\frac{(n_{ij}-n\hat{p}_{i,j})^2}{n\hat{p}_{i,j}}$
OUTS, PSY	11	$61 \cdot 59/207^2$	17.386	2.346
OUTS, MED	28	$61 \cdot 74/207^2$	21.807	1.759
OUTS, PHA	22	$61 \cdot 74/207^2$	21.807	0.002
VG, PSY	20	$84 \cdot 59/207^2$	23.942	0.649
VG, MED	34	$84 \cdot 74/207^2$	30.029	0.525
VG, PHA	30	$84 \cdot 74/207^2$	30.029	0.000
G, PSY	22	$43 \cdot 59/207^2$	12.256	7.747
G, MED	8	$43 \cdot 74/207^2$	15.372	3.535
G, PHA	13	$43 \cdot 74/207^2$	15.372	0.366
F, PSY	6	$19 \cdot 59/207^2$	5.415	0.063
F, MED	4	$19 \cdot 74/207^2$	6.792	1.148
F, PHA	9	$19 \cdot 74/207^2$	6.792	0.718
	207	1	207	$z = 18.858$

Under the null hypothesis of independence, the test statistic $\sum_{i,j} \frac{(n_{ij}-n\hat{p}_{i,j})^2}{n\hat{p}_{i,j}} \sim \chi^2_{(k'-1)(k''-1)}$, where k' is the number of categories in which the grade scale for the Statistics course has been divided ($k' = 4$) and k'' is the number of categories in which the field of study variable has been divided ($k'' = 3$).

The decision rule is to reject the null hypothesis, at the approximate 5% significance level if:

$$z > \chi^2_{\overline{(4-1),(3-1)}|0.05} = \chi^2_{6|0.05}.$$

In this case:

$$z = 18.858 > 12.60 = \chi^2_{6|0.05},$$

so that, at the 5% significance level, the null hypothesis of independence is rejected . That is, we can state that grades obtained in the Statistics course are not statistically independent from the students' field of study.