BUSINESS STATISTICS - Second Year

January 18, 2006

Academic Year 2005-06

INSTRUCTIONS

1. The exam contains multiple choice questions that must be answered in the orange code sheet we have provided you with, together with three exercises that must be answered in detail in separate sheets of paper.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions-part of the exam to copy them into the code sheet.
3. In the multiple choice questions-part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will penalize your grade by 0.2 points. Questions that have not been answered do not penalize your grade in any form.
4. Each one of the exercises, A, B and C, must be answered in a separate sheet of paper. We will collect the different parts of the exam at the indicated times and in this order: first, we will collect the code sheet for the multiple choice questions part of the exam and, then, and in this order, exercises A, B and C will be collected.
5. The exam has six numbered sheets, going from 0.1 to 0.6. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as is illustrated in the example.
6. The maximum final grade for each of the parts of the exam (i.e., the multiple choice questions part and the exercises part) is 30 points. You will need to obtain 15 points in each part of the exam to pass it. However, exams having a multiple choice question part with grades greater than or equal to 14 could, under special circumstances, be compensated with a good grade in the exercises part of the exam.
7. Please fill in your personal information in the appropriate places both in the code sheet and in the sheets provided for the exercises. In "Resit" (column II) you will write the number of times you have registered for an exam in this course *not including this one.*

Example:

$\underbrace{\text{12545}}$ PEREZ, Ernesto Exam type $\underbrace{\;0\;}$ Resit

MULTIPLE CHOICE QUESTIONS (Time: 1 hour and 30 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) Londres (E) Pekin

2. Let $\{X_n\}_{n\in\mathcal{N}}$ be a sequence of random variables having a $U[5, 5+\frac{1}{n}]$ distribution. The sequence of random variables will converge:

   (A) Only in distribution to $X = 5$

   (B) In distribution and probability to $X = 0$

   (C) In distribution and probability to $X = 5$

   (D) Only in probability to $X = 5$

   (E) All false

3. Let $X_1, \cdots, X_{400}$ be $n = 400$ independent and identically $\gamma(a = 2, r = 4)$ distributed random variables. The approximate probability that the random variable $Z = \sum_{i=1}^{400} X_i$ takes values on the interval $[780, 820]$ is:

   (A) 0.8413 (B) 1 (C) 0.6826 (D) 0 (E) All false

Questions 4 to 6 refer to the following exercise:

Let $X_1 \in N(4, \sigma^2 = 9), X_2 \in N(2, \sigma^2 = 9), X_3 \in N(0, 1)$ and $X_4 \in \chi^2_2$ be four independent r.v.

4. If we define the random variable $V = \frac{1}{9}[(X_1 - 4)^2 + (X_2 - 2)^2]$, then $P(V < 2.77)$ is:

   (A) 0.25 (B) 0.125 (C) All false (D) 0.375 (E) 0.75

5. If we define the random variable $W = \dfrac{\sqrt{2}X_3}{\sqrt{V}}$, then $P(-1.89 < W < 2.92)$ is:

   (A) 0.30 (B) 0.70 (C) 0.85 (D) 0.15 (E) All false

6. If we define the random variable $Z = \dfrac{X_4}{V}$, then the value of k such that $P(Z > k) = 0.95$ is:

   (A) 19 (B) 9 (C) All false (D) $\frac{1}{9}$ (E) $\frac{1}{19}$

Questions 7 and 8 refer to the following exercise:

It is known that in the "special sales" section of a department store the probability that goods or commodities that arrive daily from other sections have some type of defect is 0.30. We assume independence between the different goods.

7. If in a given day 20 different goods arrive at the "special sales" section of the department store, the probability that exactly 10 of them have some type of defect is:

   (A) 0.9829 (B) 0.9691 (C) 0.0309 (D) 0.0171 (E) All false

8. If in a given month, 700 goods arrive at the "special sales" section of the department store, the approximate probability that more than 200 of them have some type of defect is:

   (A) 0.7823 (B) 0.4207 (C) 0.2742 (D) 0.8918 (E) 0.5793

Questions 9 to 11 refer to the following exercise:

The number of complaints that are presented daily in the customer service section of a store follows a Poisson distribution with parameter $\lambda = 9$. We assume independence between the different complaints.

9. The approximate probability that, on a given day, more than 7 complaints are presented in the customer service section of the store is:

   (A) 0.2068 (B) 0.3239 (C) 0.7932 (D) 0.6761 (E) 0.5443

10. The approximate probability that during a given week (5 workable days) less than 40 complaints are presented in the customer service section of the store is:

    (A) 0.7939 (B) 0.4703 (C) 0.4522 (D) 0.5478 (E) 0.2061

11. The approximate probability that in a given week (5 workable days) there is only 1 day in which more than 7 complaints are presented in the customer service section of the store is:

    (A) 0.007 (B) 0.3384 (C) 0.0372 (D) 0.6616 (E) 0.9930

Questions 12 and 13 refer to the following exercise:

Let X be a discrete r.v. with probability mass function given by:

$$P(X=0)=\theta \qquad P(X=1)=P(X=2)=2\theta \qquad P(X=3)=1-5\theta$$

In order to estimate the parameter θ, a random sample of size $n = 10$ has been taken, providing the following result: 0, 1, 1, 2, 2, 3, 3, 3, 3, 3.

12. The method of moments estimate of θ is:

    (A) 0.10 (B) 0.50 (C) 0.20 (D) 0.25 (E) 0.15

13. The maximum likelihood estimate of θ is:

    (A) 0.15 (B) 0.50 (C) 0.10 (D) 0.25 (E) 0.20

Questions 14 to 17 refer to the following exercise:

Let X be a r.v. with probability density function given by: $f(x,\theta) = \dfrac{3}{\theta^3}\, x^2, \quad x \in [0,\theta]$, and let $X_1, X_2, \cdots, X_n$ be a random sample from this distribution. We know that the mean of this random variable is $m = \frac{3\theta}{4}$ and that its variance is $\sigma^2 = \frac{3\theta^2}{80}$.

14. The method of moments estimator of θ, $\hat{\theta}_{\text{MM}}$, is:

    (A) $\frac{3\overline{X}}{4}$ (B) All false (C) $\max\{X_i\}$ (D) $\min\{X_i\}$ (E) $\frac{4\overline{X}}{3}$

15. Is $\hat{\theta}_{\text{MM}}$ an unbiased estimator of θ?

    (A) No (B) It cannot be established (C) Yes (D) - (E) -

16. Is $\hat{\theta}_{\text{MM}}$ a consistent estimator of θ?

    (A) Yes (B) It cannot be established (C) - (D) - (E) No

17. The maximum likelihood estimator of θ, $\hat{\theta}_{\text{ML}}$, is:

(A) $\max\{X_i\}$ (B) $\frac{3\overline{X}}{4}$ (C) $\min\{X_i\}$ (D) $\frac{4\overline{X}}{3}$ (E) All false

18. Let X be a r.v. having a Poisson distribution with parameter λ. Based on a random sample of size n, we consider the following estimator for λ: $\hat{\lambda} = \dfrac{X_1 + \cdots + X_n}{k}$. What is the value that k should take so that $\hat{\lambda}$ is an unbiased estimator of λ?

(A) n (B) $\dfrac{1}{n}$ (C) $\dfrac{1}{\overline{x}}$ (D) All false (E) $\overline{x}$

Questions 19 and 20 refer to the following exercise:

Let X be a random variable with probability density function given by:

$$f(x, \theta) = 3^\theta \; x^{\theta-1} e^{-3x} \qquad x > 0, \qquad \theta > 0$$

In order to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$, a random sample of size $n = 1$ has been taken.

19. For a given significance level, the most powerful critical region for this test is of the form:

(A) $X \geq C$ (B) $X \in (C_1, C_2)^c$ (C) $X \leq C$ (D) $X \in (C_1, C_2)$ (E) All false

20. At the $\alpha = 5\%$ significance level, we reject H_0 if:

(A) $X \leq 0.9986$ (B) $X \in (0.0171, 0.998)$ (C) $X \geq 0.0171$ (D) $X \leq 0.0171$ (E) $X \geq 0.9986$

Questions 21 and 22 refer to the following exercise:

Let X be a random variable having a Poisson distribution with parameter λ. In order to test the null hypothesis $H_0 : \lambda \leq 2$ against the alternative hypothesis $H_1 : \lambda > 2$, a random sample of size $n = 3$ has been taken. We use $T = \sum_{i=1}^{3} X_i$ as the test statistic and decide to reject H_0 if $T > 10$.

21. The approximate significance level for this test is:

(A) 0.084 (B) 0.043 (C) 0.050 (D) 0.020 (E) 0.112

22. For $\lambda = 3$, the power for this test is:

(A) 0.8030 (B) 0.2940 (C) 0.4126 (D) 0.1970 (E) 0.7060

Questions 23 to 25 refer to the following exercise:

A carpentry store considers that opening a new branch in a nearby region is profitable if at least 40% of the inhabitants in that region show some interest in making use of its services. In order to test the null hypothesis $H_0 : p \geq 0.40$ against the alternative hypothesis $H_1 : p < 0.40$, a random sample of $n = 20$ inhabitants of that region has been taken.

23. Let Z be the random variable that represents the number of inhabitants of that region, from the selected 20 inhabitants, which shows some interest in making use of its services. At the $\alpha = 5\%$ significance level, the store will not open a new branch in the nearby region if:

(A) $Z \leq 4$ (B) $Z < 4$ (C) $Z \geq 4$ (D) $Z > 4$ (E) $Z > 2$

24. If $p = 0.30$, the power for this test is:

(A) 0.1071 (B) 0.8929 (C) 0.950 (D) 0.7625 (E) 0.2375

25. If there are 8 inhabitants in the sample that have shown some interest in making use of its services, the decision the carpentry store will take is:

(A) Reject H_0 (B) - (C) Do not reject H_0 (D) - (E) -

Questions 26 and 27 refer to the following exercise:

We wish to estimate the proportion, p, of cyclists that make frequent use of the cycle-path when going out in their bikes.

26. If we have taken a random sample of 800 cyclists and have obtained that 300 of them make frequent use of the cycle-path, a 95% confidence interval for the proportion p is, approximately:

(A) $(0.3415,\ 0.4085)$ (B) $(0.3239,\ 0.5739)$ (C) $(0.2575,\ 0.7250)$ (D) $(0.3750,\ 0.6750)$ (E) $(0.5915,\ 0.6585)$

27. We wish to test the null hypothesis $H_0 : p = 0.35$ against the alternative hypothesis $H_1 : p \neq 0.35$. At the $\alpha = 5\%$ significance level, the decision of this test will be:

(A) Do not reject H_0 (B) - (C) Reject H_0 (D) - (E) -

28. It is known that the expenses a given population makes on pharmaceutical products follows a normal distribution. Researchers have the hypothesis that the variance of such expenses made by individuals who are younger than 50, σ_1^2, is greater than or equal than the variance of the expenses made by older individuals, σ_2^2. In order to test the null hypothesis $H_0 : \sigma_1^2 \geq \sigma_2^2$ against the alternative hypothesis $H_1 : \sigma_1^2 < \sigma_2^2$, two independent random samples of equal sizes $n_1 = n_2 = 31$ have been taken, providing the sample variances $s_1^2 = 62$ and $s_2^2 = 65$. At the $\alpha = 5\%$ significance level, the decision of the test will be:

(A) Reject H_0 (B) - (C) - (D) - (E) Do not reject H_0

Questions 29 and 30 refer to the following exercise:

Let X and Y be two independent r.v. such that $X \in N(m_X, \sigma_X^2 = 25)$ and $Y \in N(m_Y, \sigma_Y^2 = 36)$. In order to test the null hypothesis $H_0 : m_X = m_Y$ against the alternative hypothesis $H_1 : m_X \neq m_Y$, two independent random samples of equal sizes $n_X = n_Y = 30$ were taken, providing the sample means $\overline{x} = 82$ and $\overline{y} = 80$.

29. A 90% confidence interval for $(m_X - m_Y)$ is, approximately:

(A) $(-0.7949,\ 4.7949)$ (B) $(-1.3347,\ 5.3347)$ (C) $(-0.3386,\ 4.3386)$ (D) $(0.3386,\ 4.3386)$ (E) $(0.7949,\ 4.7949)$

30. At the $\alpha = 10\%$ significance level, the decision of the test will be:

(A) Reject H_0 (B) - (C) - (D) - (E) Do not reject H_0

EXERCISES (Time: 75 minutes)

A. (10 points, 25 minutes)

Let X be a random variable with probability density function given by:

$$f(x,\theta) = \frac{1}{6\theta^4} x^3 e^{-\frac{1}{\theta}x} \qquad x > 0, \qquad \theta > 0$$

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \cdots, X_n$, has been taken.

i) Find, **providing all relevant details**, the maximum likelihood estimator of the parameter θ.

ii) Find, **providing all relevant details**, the method of moments estimator of the parameter θ.

iii) Is the maximum likelihood estimator of θ unbiased? consistent? efficient?

Hint: Note that if $X \in \gamma(a,r)$, then $f(x,a,r) = \frac{a^r}{\Gamma(r)} x^{r-1} e^{-ax}$, $x > 0$, $a, r > 0$. In addition, note that the equation for the Cramer-Rao lower bound for the variance of a regular unbiased estimator from a random sample is $L_c = \frac{1}{n\mathrm{E}\left(\frac{\partial \ln(f(x,\theta))}{\partial \theta}\right)^2}$.

B. (10 points, 25 minutes)

The sales price, in thousands of euros, of second homes in a given region follows a normal $N(m, \sigma^2)$ distribution. In order to test the null hypothesis $H_0 : m \leq 337$ against the alternative hypothesis $H_1 : m > 337$, a random sample of $n = 26$ second homes on sale was taken, providing the sample mean and variance $\overline{x} = 340$ and $s^2 = 150$.

i) At the $\alpha = 5\%$ significance level, test the previously specified hypotheses.

ii) Find a 95% confidence interval for the population variance σ^2.

C. (10 points, 25 minutes)

In a hospital, researchers believe that the weight newborn babies have (in Kilograms) follows a normal $N(m = 3.30, \sigma^2 = 0.5)$ distribution. In order to test this hypothesis, a random sample of $n = 200$ newborn babies was taken, providing the following results:

Weight	Less than 2 Kg.	2 to 3 Kg.	3 to 4 Kg.	4 to 5 Kg.	More than 5 Kg.
Newborn babies	5	60	104	29	2

Using the information provided by the sample and at the $\alpha = 5\%$ significance level, test the previously specified hypothesis.

SOLUTIONS TO MULTIPLE CHOICE QUESTIONS

1: C	11: C	21: B
2: C	12: A	22: B
3: C	13: C	23: B
4: E	14: E	24: A
5: C	15: C	25: C
6: E	16: A	26: A
7: C	17: A	27: A
8: A	18: A	28: E
9: D	19: A	29: C
10: E	20: E	30: E

SOLUTIONS TO EXERCISES

A)

The probability density function for the random variable X is given by:

$$f(x, \theta) = \frac{1}{6\theta^4} x^3 e^{-\frac{1}{\theta}x} \qquad x > 0, \qquad \theta > 0$$

Thus, making use of the hint provided in this exercise, it is straightforward to verify that:

$$X \in \gamma\left(a = \frac{1}{\theta}, r = 4\right)$$

From this, we have that $\mathrm{E}(X) = \frac{r}{a} = 4\theta$ and that $\mathrm{Var}(X) = \frac{r}{a^2} = 4\theta^2$.

i) Method of moments estimator

In order to obtain this estimator, we equate the first population moment to the first sample moment. That is,

$$\alpha_1 = \mathrm{E}(X) = a_1 = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X}$$

As we already know, $\mathrm{E}(X) = 4\theta$, so that:

$$\alpha_1 = 4\theta = \frac{\sum_{i=1}^{n} X_i}{n} = \overline{X} = a_1$$

and, therefore,

$$\hat{\theta}_{MM} = \frac{\overline{X}}{4}$$

ii) Maximum likelihood estimator

The likelihood function for the sample will be given by:

$$\begin{aligned} L(\theta) &= f(x_1; \theta) \ \ldots \ f(x_n; \theta) \\ &= \left[\frac{1}{6\theta^4} x_1^3 e^{-\frac{1}{\theta}x_1}\right] \ \ldots \ \left[\frac{1}{6\theta^4} x_n^3 e^{-\frac{1}{\theta}x_n}\right] \\ &= \frac{1}{6^n \theta^{4n}} \left[\prod_{i=1}^{n} x_i^3\right] e^{-\frac{1}{\theta}\sum_{i=1}^{n} x_i} \end{aligned}$$

We now compute its natural logarithm to obtain:

$$\ln L(\theta) = -n \ln 6 - 4n \ln \theta + \ln\left[\prod_{i=1}^{n} x_i^3\right] - \frac{1}{\theta}\sum_{i=1}^{n} x_i$$

If we take derivatives with respect to θ and equate this to zero, we will have that:

$$\frac{\partial \ln L(\theta)}{\partial \theta} = \frac{-4n}{\theta} + \frac{1}{\theta^2}\sum_{i=1}^{n} x_i = 0$$

from which, we obtain that,

$$-4n\theta + \sum_{i=1}^{n} x_i = 0$$

$$\hat{\theta}_{ML} = \frac{\sum_{i=1}^{n} X_i}{4n} = \frac{\overline{X}}{4}$$

iii) Unbiasedness:

In order to check if the estimator is unbiased, we have to verify if $\mathrm{E}(\hat{\theta}_{ML}) = \theta$.

In this case,

$$\mathrm{E}(\hat{\theta}_{ML}) = \mathrm{E}\left(\frac{1}{4}\,\overline{X}\right) = \frac{1}{4}\,\mathrm{E}(X) = \frac{1}{4}\,(4\theta) = \theta$$

Therefore, it is an unbiased estimator of θ.

Consistency

In order to check if the estimator is consistent, we need to compute its variance.

$$\mathrm{Var}(\hat{\theta}_{ML}) = \mathrm{Var}\left(\frac{1}{4}\,\overline{X}\right) = \frac{1}{16}\,\frac{\mathrm{Var}(X)}{n} = \left(\frac{1}{16}\right)\,\left(\frac{4\theta^2}{n}\right) = \frac{\theta^2}{4n}$$

Given that it is an unbiased estimator whose variance goes to zero as n goes to infinity, we have that the two sufficient conditions for consistency hold and, therefore, it is a consistent estimator of θ.

Efficiency

In order to check if the estimator is efficient, we have to verify if its variance coincides with the Cramer-Rao lower bound for a regular and unbiased estimator of θ.

By making use of the hint provided in the exercise, the Cramer-Rao lower bound is given by:

$$L_c = \frac{1}{n\mathrm{E}\left(\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right)^2}$$

Therefore, to compute this bound we need to proceed as follows:

$$\ln(f(x,\theta)) = -\ln 6 - 4\ln\theta + 3\ln x - \frac{1}{\theta}x$$

$$\frac{\partial \ln(f(x,\theta))}{\partial\theta} = -\frac{4}{\theta} + \frac{x}{\theta^2} = \frac{1}{\theta^2}(x - 4\theta)$$

$$\left(\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right)^2 = \frac{1}{\theta^4}(x - 4\theta)^2$$

$$\mathrm{E}\left(\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right)^2 = \frac{1}{\theta^4}\mathrm{E}(X - 4\theta)^2 = \frac{1}{\theta^4}\mathrm{Var}X = \frac{4\theta^2}{\theta^4} = \frac{4}{\theta^2}$$

$$n\mathrm{E}\left(\frac{\partial \ln(f(x,\theta))}{\partial\theta}\right)^2 = \frac{4n}{\theta^2}$$

and

$$L_c = \frac{\theta^2}{4n} = \mathrm{Var}(\hat{\theta}_{ML})$$

Thus, $\hat{\theta}_{ML}$ is an efficient estimator of θ.

B)

Let X be a r.v. representing the sales price, in thousands of euros, of second homes in a given region, so that $X \in N(m, \sigma^2)$. We wish to test the null hypothesis $H_0 : m \leq 337$ against the alternative hypothesis $H_1 : m > 337$. In order to do so, a random sample of size $n = 26$ has been taken, providing that $\overline{x} = 340$ and $s^2 = 150$.

i) Given that σ^2 is unknown, the test statistic and its distribution under the null hypothesis H_0 will be:

$$\left(\frac{\overline{X} - m_0}{\frac{S}{\sqrt{(n-1)}}} \right) \in t_{(n-1)},$$

so that, at the α significance level, the null hypothesis H_0 is rejected if:

$$\left(\frac{\overline{x} - m_0}{\frac{s}{\sqrt{(n-1)}}} \right) \geq t_{(n-1);\alpha}$$

In this case, we have that $\dfrac{340 - 337}{\frac{\sqrt{150}}{\sqrt{25}}} = 1.22 < t_{25;\alpha=0.05} = 1.71$, so that, at the $\alpha = 5\%$ significance level, we do not reject the null hypothesis $H_0 : m \leq 337$.

ii) In order to obtain the confidence interval to estimate σ^2 we use the statistic: $\dfrac{nS^2}{\sigma^2} \in \chi^2_{(n-1)}$, that provides the following $(1-\alpha)\%$ confidence interval:

$$\left[\frac{ns^2}{\chi^2_{(n-1);\frac{\alpha}{2}}} ; \frac{ns^2}{\chi^2_{(n-1);1-\frac{\alpha}{2}}} \right]$$

More specifically, and using the data provided by the sample, the 95% confidence interval for the variance σ^2 of the random variable X is:

$$\left[\frac{26 \times 150}{\chi^2_{25;0.025}} ; \frac{26 \times 150}{\chi^2_{25;0.975}} \right] == \left[\frac{26 \times 150}{40.6} ; \frac{26 \times 150}{13.1} \right] = [96.06; 297.71]$$

C)

Let X be the random variable representing the weight (in Kilograms) newborn babies have.

We wish to test the null hypothesis $H_0 : X \in N(m = 3.30, \sigma^2 = 0.5)$ against the alternative hypothesis $H_1 : X \notin N(m = 3.30, \sigma^2 = 0.5)$. In order to do this, a random sample of $n = 200$ newborn babies has been taken.

Therefore, this is a χ^2 goodness of fit test to a completely specified distribution.

In order to carry out this test, we need to compute the corresponding test statistic and, thus, we build the following table:

Class	n_i	P_i	nP_i	$\frac{(n_i - nP_i)^2}{nP_i}$
< 2	5	0.0329	6.58	
$2 - 3$	60	0.3043	60.86	
$3 - 4$	104	0.4993	99.86	
$4 - 5$	29	0.1553	31.06	
> 5	2	0.0082	1.64	
	200	1	200	

where the corresponding probabilities P_i have been calculated by using the distribution of the random variable X under the null hypothesis H_0: $X \in N(3.30, 0.5)$. That is,

$$P(X < 2) = \Phi\left(\frac{2 - 3.30}{\sqrt{0.50}}\right) = \Phi(-1.84) = 1 - 0.9671 = 0.0329$$

$$P(2 < X < 3) = \Phi\left(\frac{3 - 3.30}{\sqrt{0.50}}\right) - \Phi\left(\frac{2 - 3.30}{\sqrt{0.50}}\right) = \Phi(-0.42) - 0.0329 = (1 - 0.6628) - 0.0329 = 0.3043,$$

and so on.

Remark: Given that we have not used interpolation from the corresponding table, the values obtained for these probabilities are only approximated values.

In the table above, we can see that there is a class, $(X > 5)$, which has a theoretical frequency smaller than 5, so it is necessary to group this class with the class right before it. Therefore, the new table that will allow us to find the value for the test statistic will be given by:

Class	n_i	P_i	nP_i	$\frac{(n_i - nP_i)^2}{nP_i}$
< 2	5	0.0329	6.58	0.37939
$2 - 3$	60	0.3043	60.86	0.01215
$3 - 4$	104	0.4993	99.86	0.17163
> 4	31	0.1635	32.70	0.08838
	200	1	200	$z = 0.65155$

Under H_0, the test statistic $Z = \displaystyle\sum_{i=1}^{k} \frac{(n_i - nP_i)^2}{nP_i} \in \chi^2_{k-1}$, where $k = 4$ is the number of classes in the final table.

The decision rule indicates that the null hypothesis should be rejected if $z > \chi^2_{3;0.05}$, where z is the sample value of the test statistic Z.

In this case, $z = 0.65155 < \chi^2_{3;0.05} = 7.81$, so that, at the $\alpha = 5\%$, we do not reject the null hypothesis $H_0 : X \in N(3.30, 0.50)$.