INSTRUCTIONS

1. The quiz contains multiple choice questions that must be answered in the orange code sheet we have provided you with.

2. To select an answer, all you need to do is mark in the orange code sheet, filling the rectangle over which the selected answer is located appropriately. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.

3. In the multiple choice questions part of the exam there is always only one correct answer for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will not penalize your grade at all. Questions that have not been answered do not penalize your grade in any form.

4. The quiz has three numbered sheets, going from 0.1 to 0.3. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.

5. The maximum final grade is 14 points. You will need to obtain 10 points to pass this quiz.

7. Please fill in your personal information in the appropriate places in the code sheet.

Example: 12545 PEREZ, Ernesto Exam type 0 Resit
MULTIPLE CHOICE QUESTIONS (Time: 40 minutes)

1. FREE-QUESTION. The capital of Spain is:
   (A) Paris  (B) Sebastopol  (C) Madrid  (D) London  (E) Pekin

Questions 2 and 4 refer to the following exercise:

In a given community, the probability that a family taken at random within that community has a private medical insurance policy is 0.4. We assume independence between the different families in that community.

2. If we ask 8 randomly selected families, the approximate probability that 7 of them have a private medical insurance policy is:
   (A) 0.0112  (B) 0.0510  (C) 0.1427  (D) 0.0280  (E) 0.0079

3. If we now ask 20 randomly selected families, the probability that at least 12 of them have a private medical insurance policy is:
   (A) 0.8744  (B) 0.9435  (C) 0.0565  (D) 0.9997  (E) 0.0003

4. If we now ask 200 randomly selected families, what is the approximate probability that at most 90 of them have a private medical insurance policy?
   (A) 0.0643  (B) 0.9357  (C) 0.8554  (D) 0.3372  (E) 0.6628

Questions 5 to 7 refer to the following exercise:

The number of clients arriving each hour at a given branch of a bank asking for a given service follows a Poisson distribution with parameter $\lambda = 3$. It is assumed that arrivals at different hours are independent from each other.

5. The probability that in a given hour at most 4 clients arrive at this specific branch of the bank is:
   (A) 0.6472  (B) 0.9161  (C) 0.8153  (D) 0.1847  (E) 0.0839

6. The approximate probability that during a regular working day of eight hours at least 25 clients arrive at this specific branch of the bank is:
   (A) 0.9362  (B) 0.5398  (C) 0.3821  (D) 0.6179  (E) 0.4602

7. If in a regular working day (8 hours) the branch is able to provide service to a maximum of 29 clients, what is the approximate probability that all of the clients arriving that day asking for that service are properly handled by the branch?
   (A) 0.1314  (B) 0.8212  (C) 0.1788  (D) 0.8686  (E) 0.3446
8. Let \( \{X_n\}_{n \in \mathbb{N}} \) be a sequence of random variables having probability mass function given by: \( P(X_n = 0) = 1 - \frac{1}{n} \), \( P(X_n = n^3) = \frac{1}{n} \). The sequence will converge to the r.v. degenerate at 0,
   (A) only in distribution and probability.
   (B) only in quadratic mean.
   (C) only in probability.
   (D) only in distribution.
   (E) in distribution, probability and quadratic mean.

9. Let \( \{X_n\}_{n \in \mathbb{N}} \) be a sequence of r.v. having characteristic function given by \( \psi_n(u) = e^{4iu - \frac{2u^2}{n}} \). The sequence will converge:
   (A) Only in distribution to \( X = 4 \).
   (B) Only in probability to \( X = 4 \).
   (C) Only in distribution to \( X = 2 \).
   (D) In probability and distribution to \( X = 2 \).
   (E) In probability and distribution to \( X = 4 \).

10. Let \( X_1 \) and \( X_2 \) be independent r.v. such that \( X_1 \in \gamma(1, 2) \) and \( X_2 \in \gamma(2, 1) \). If we let \( Y = 2X_1 + 4X_2 \), then the distribution of the r.v. \( Y \) is:
    (A) \( \gamma(0.5, 3) \)  \quad (B) \( \chi^2_3 \)  \quad (C) \( \gamma(0.25, 3) \)  \quad (D) \( \chi^2_{12} \)  \quad (E) \( \gamma(2, 3) \)

11. The lifetime of a home appliance (in thousands of hours) follows an exponential distribution with mean \( m = 2 \). The manufacturer agrees to replace the home appliance if it fails before its 500 hours warranty period expires. What is the approximate probability that the manufacturer is required to replace a given home appliance?
    (A) 0.2212  \quad (B) 0.7788  \quad (C) 0.4217  \quad (D) 0.3728  \quad (E) 0.1423

12. Let \( X_1 \) and \( X_2 \) be two independent and identically distributed r.v. having a \( \chi^2_{10} \) distribution. The value of \( k \) such that \( P(X_1 + X_2 \leq k) = 0.75 \) is:
    (A) 23.8  \quad (B) 10.9  \quad (C) 19.3  \quad (D) 28.4  \quad (E) 15.5

Questions 13 and 14 refer to the following exercise:

Let \( X_1, X_2, X_3, X_4 \) and \( X_5 \) be independent r.v. such that: \( X_1, X_2 \) and \( X_3 \in N(3, \sigma^2 = 4) \), \( X_4 \) and \( X_5 \in N(5, \sigma^2 = 4) \).

13. If we let \( Y = \frac{2 \frac{(X_1 - 3)}{2}}{\sqrt{(X_2 - 3)^2 + (X_3 - 3)^2 + (X_4 - 5)^2 + (X_5 - 5)^2}} \) then we have that \( P(Y \leq 1.53) \) is:
    (A) 0.2  \quad (B) 0.4  \quad (C) 0.1  \quad (D) 0.8  \quad (E) 0.9

14. If we let \( Z = \frac{3 \left((X_4 - 5)^2 + (X_5 - 5)^2\right)}{2 \left((X_1 - 3)^2 + (X_2 - 3)^2 + (X_3 - 3)^2\right)} \) then the value of \( k \) such that \( P(Z \leq k) = 0.9 \) is:
    (A) 5.46  \quad (B) 30.8  \quad (C) 1/9.16  \quad (D) 9.16  \quad (E) 9.55

- 0.3 -