BUSINESS STATISTICS - Second Year
Voluntary Quiz 2

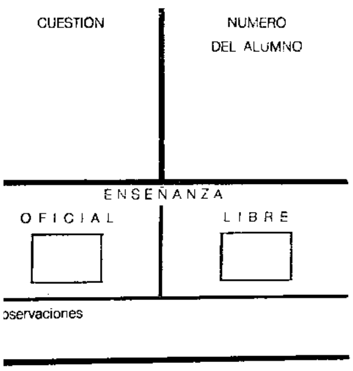
Academic Year 2007-08
January 10, 2008

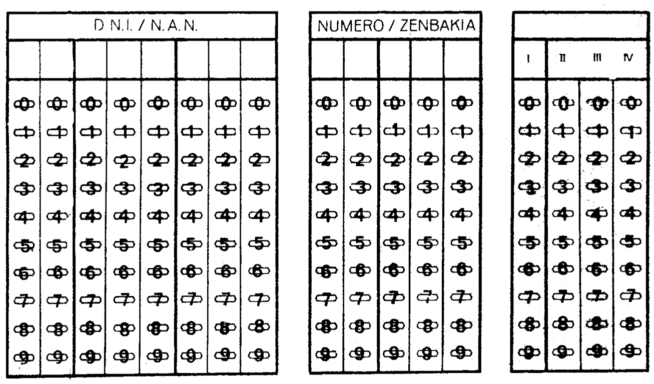
INSTRUCTIONS

1. The quiz contains multiple choice questions that must be answered in the orange code sheet we have provided you with.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.
3. In the multiple choice questions part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will not penalize your grade at all. Questions that have not been answered do not penalize your grade in any form.
4. The quiz has four numbered sheets, going from 0.1 to 0.4. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 0. Mark a 0 in the column labelled with I in your code sheet, just as it is illustrated in the example.
5. The maximum final grade is 14 points. **You will need to obtain 10 points to pass this quiz**.
7. Please fill in your personal information in the appropriate places in the code sheet.

Example:

$\underbrace{12545}$ PEREZ, Ernesto Exam type $\underbrace{0}$ Resit





MULTIPLE CHOICE QUESTIONS (Time: 50 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Paris (B) Sebastopol (C) Madrid (D) Londres (E) Pekin

Questions 2 to 4 refer to the following exercise:

Let X be a r.v. with probability density function given by

$$f(x,\theta) = \begin{cases} \dfrac{2x}{\theta^2}\ e^{-\left(\frac{x}{\theta}\right)^2} & \text{for } x \geq 0,\ \ \theta > 0; \\ 0 & \text{otherwise,} \end{cases}$$

and it is known that the mean of X is $m = \theta\ \Gamma(3/2)$.

We wish to estimate the parameter θ and, in order to do so, a random sample of size n, X_1, X_2, $\cdots$, X_n, has been taken.

2. The method of moments estimator of θ, $\hat{\theta}_{MM}$, is:

   (A) $\dfrac{\overline{X}}{\Gamma(3/2)}$ (B) $\dfrac{1}{\overline{X}}$ (C) $2\overline{X}$ (D) $\overline{X} - 1$ (E) $\overline{X}\ \Gamma(3/2)$

3. Is $\hat{\theta}_{MM}$ an unbiased estimator of θ?

   (A) - (B) - (C) Yes (D) - (E) No

4. The maximum likelihood estimator of θ, $\hat{\theta}_{ML}$, is:

   (A) $\overline{X}$ (B) $\dfrac{n}{\sum_{i=1}^n X_i^2}$ (C) $\left(\dfrac{n}{\sum_{i=1}^n X_i^2}\right)^{1/2}$ (D) $\left(\dfrac{\sum_{i=1}^n X_i^2}{n}\right)^{1/2}$ (E) $\dfrac{\sum_{i=1}^n X_i^2}{n}$

Questions 5 and 6 refer to the following exercise:

Let X be a r.v. with probability mass function given by: $P(X = -2) = \frac{\theta}{2}$, $P(X = 0) = 1 - \theta$, $P(X = 2) = \frac{\theta}{2}$. In order to estimate the parameter θ, a random sample of size $n = 10$ has been taken, providing the following results: -2, -2, -2, 0, 0, 0, 0, 2, 2, 2.

5. The method of moments estimate of θ is:

   (A) 0.50 (B) 0.60 (C) 0.40 (D) 2.40 (E) 0

6. The maximum likelihood estimate of θ is:

   (A) 0.40 (B) 0.60 (C) 0 (D) 2.40 (E) 0.50

Questions 7 to 9 refer to the following exercise:

Let $X_1, \ldots, X_n$ be a random sample taken from a $N(0, \theta)$ population, where θ is the distributional variance.

7. A sufficient statistic for θ is:

   (A) $\displaystyle\sum_{i=1}^n X_i^2$ (B) $\displaystyle\prod_{i=1}^n X_i$ (C) $\displaystyle\sum_{i=1}^n X_i$ (D) $\displaystyle\prod_{i=1}^n \ln(X_i)$ (E) $\displaystyle\prod_{i=1}^n X_i^2$

8. The method of moments estimator of θ, $\hat{\theta}_{MM}$, is:

(A) $\overline{X}$ (B) $\dfrac{1}{\sum_{i=1}^n X_i^2}$ (C) $\left(\overline{X}\right)^{1/2}$ (D) $\dfrac{1}{\overline{X}}$ (E) $\dfrac{\sum_{i=1}^n X_i^2}{n}$

9. The maximum likelihood estimator of θ, $\hat{\theta}_{ML}$, is:

(A) $\dfrac{\sum_{i=1}^n X_i^2}{n}$ (B) $\dfrac{1}{\sum_{i=1}^n X_i^2}$ (C) $\overline{X}$ (D) $\dfrac{1}{\overline{X}}$ (E) $\left(\overline{X}\right)^{1/2}$

Questions 10 and 11 refer to the following exercise:

Let $X_1, \ldots, X_n$ be a r.s. taken from a $U[0, \theta+1]$ population (having a uniform distribution between 0 and $\theta+1$). If we define the estimators

$$\hat{\theta}_1 = 2\overline{X} - 1$$

$$\hat{\theta}_2 = \frac{X_1 + 2X_2 + \cdots + 2X_{n-1} + X_n}{(n-1)} - 1$$

10. We have that:
   - (A) Only $\hat{\theta}_2$ is unbiased
   - (B) Only $\hat{\theta}_1$ is unbiased
   - (C) Both estimators are biased
   - (D) Both estimators are unbiased
   - (E) It cannot be determined from the information provided

11. We have that:
   - (A) Only $\hat{\theta}_2$ is consistent
   - (B) Only $\hat{\theta}_1$ is consistent
   - (C) -
   - (D) Both estimators are consistent
   - (E) It cannot be determined from the information provided

Questions 12 to 14 refer to the following exercise:

We wish to test the hypothesis that the probability density function of a given population is $\gamma(a=5, r)$, against the alternative hypothesis that it is $\gamma(a=1, r)$; that is, the parameter r is common to both distributions. We recall that the probability density function for a $\gamma(a, r)$ distribution is given by:

$$f(x; a, r) = \frac{a^r}{\Gamma(r)}\, x^{r-1} e^{-ax}, \quad x > 0,\ a, r > 0$$

In order to carry out this test, a random sample of size $n = 1$ has been taken from that population (that is, we observe X).

12. The most powerful critical region for X is of the form:

(A) $[K, +\infty]$ (B) $[-\infty, K]$ (C) All false (D) $[K_1, K_2]^c$ (E) $[K_1, K_2]$

13. If $r = 1$ and $x = 0.48$, what would be the decision at the 0.05 significance level?

(A) - (B) Do not reject H_0 (C) - (D) - (E) Reject H_0

14. What would be the approximate power for this specific case and specific significance level?

(A) 0.55 (B) 0.45 (C) 0.21 (D) 0.32 (E) 0.68