BUSINESS STATISTICS - Second Year — Academic Year 2005-06

Voluntary Quiz 2 — December 15, 2006

INSTRUCTIONS

1. The quiz contains multiple choice questions that must be answered in the orange code sheet we have provided you with.
2. To select an answer, all you need to do is mark in the orange code sheet, **filling the rectangle over which the selected answer is located appropriately**. Please make sure you know the answer you wish to mark before doing it. Even though you can always erase your mark if you have used a pencil (number 2 or similar), any mark that has not been completely erased could be read by the machine. Therefore, we advice you to first mark your selected answers in the exam and to use only the last ten minutes or so from the time assigned to the multiple choice questions part of the exam to copy them into the code sheet.
3. In the multiple choice questions part of the exam there is always **only one correct answer** for every question. Every question correctly answered is worth 1 point, while each question incorrectly answered will not penalize your grade at all. Questions that have not been answered do not penalize your grade in any form.
4. The quiz has three numbered sheets, going from 1.1 to 1.3. Please make sure that you have all sheets and contact your professor if this is not the case. There are different exam types. This exam is of type 1. Mark a 1 in the column labelled with I in your code sheet, just as it is illustrated in the example.
5. The maximum final grade is 11 points. **You will need to obtain 8 points to pass this quiz**.

7. Please fill in your personal information in the appropriate places in the code sheet.

Example:

$\underbrace{12545}$ PEREZ, Ernesto   Exam type $\underbrace{1}$ Resit

MULTIPLE CHOICE QUESTIONS (Time: 40 minutes)

1. FREE-QUESTION. The capital of Spain is:

   (A) Madrid (B) Paris (C) Sebastopol (D) Londres (E) Pekin

Questions 2 and 3 refer to the following exercise:

Let X be a random variable with probability density function given by

$$f(x,\theta) = \begin{cases} \dfrac{1}{\theta}\, x^{-\left(\frac{1}{\theta}+1\right)} & \text{for } x \geq 1,\ \ \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

In addition, it is known that the mean of X is $m = (1-\theta)^{-1}$.

We wish to estimate the parameter θ so that, in order to do so, a random sample of size n, X_1, X_2, $\cdots$, X_n, is taken.

2. The method of moments estimator of θ, $\hat{\theta}_{MM}$, will be:

   (A) $\dfrac{\overline{X}}{\overline{X}-1}$ (B) $\dfrac{1}{\overline{X}}$ (C) $\overline{X}$ (D) $\dfrac{\overline{X}-1}{\overline{X}}$ (E) $\overline{X}-1$

3. The maximum likelihood estimator of θ, $\hat{\theta}_{MV}$, will be:

   (A) $\overline{X}$ (B) $\dfrac{\sum_{i=1}^{n}\ln(X_i)}{n}$ (C) $\dfrac{1}{\sum_{i=1}^{n}\ln(X_i)}$ (D) $\dfrac{1}{\overline{X}}$ (E) $\dfrac{\sum_{i=1}^{n}\ln(X_i)}{\overline{X}}$

Questions 4 and 5 refer to the following exercise:

Let X be a random variable with probability mass function given by: $P(X=-1) = \frac{\theta}{4}$, $P(X=0) = 1-\frac{\theta}{2}$, $P(X=1) = \frac{\theta}{4}$. In order to estimate the parameter θ, a random sample of size $n = 8$ has been taken, providing the following results: -1, -1, 0, 0, 0, 0, 1, 1.

4. The method of moments estimate of θ is:

   (A) 1 (B) $\frac{1}{4}$ (C) 0 (D) $\frac{1}{2}$ (E) 2

5. The maximum likelihood estimate of θ is:

   (A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{4}$ (D) 2 (E) 0

Questions 6 to 8 refer to the following exercise:

Let X be a random variable with probability mas function given by

$$P(x,\theta) = \begin{cases} \dfrac{e^{-\theta}\, \theta^{x}}{x!} & \text{for } x = 0, 1, \ldots,\quad \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, \ldots, X_n$, has been taken. We define the two estimators $\hat{\theta}_1$ and $\hat{\theta}_2$ as

$$\hat{\theta}_1 = \frac{X_1 + 2X_2 + \cdots + 2X_{n-1} + X_n}{(2n-2)} \text{ and } \hat{\theta}_2 = \frac{X_1 + 2X_2 + X_n}{4}.$$

6. A sufficient statistic for θ is:

(A) $\sum_{i=1}^n X_i$ (B) $\prod_{i=1}^n X_i!$ (C) $\prod_{i=1}^n X_i^2$ (D) $\prod_{i=1}^n X_i$ (E) $\sum_{i=1}^n X_i^2$

7. For these two estimators, we have that:
   - (A) Both estimators are unbiased
   - (B) Both estimators are biased
   - (C) Only $\hat{\theta}_1$ is unbiased
   - (D) Only $\hat{\theta}_2$ is unbiased
   - (E) It cannot be determined from the information provided

8. Is $\hat{\theta}_1$ a consistent estimator?

(A) No (B) – (C) It cannot be determined from the information provided (D) Yes (E) –

Questions 9 and 10 refer to the following exercise:

Let X be a random variable with probability density function given by:

$$f(x, \theta) = (\theta + 2)x^{\theta+1}, \quad 0 \leq x \leq 1, \quad \theta > 0$$

we wish to test the null hypothesis $H_0 : \theta = 1$ against the alternative hypothesis $H_1 : \theta = 2$. In order to do this, a random sample of size $n = 1$, X_1, has been taken.

9. The most powerful critical region for X_1 is of the form:

(A) $[C, 1]$ (B) $[0, C]$ (C) $[C_1, C_2]^C$ (D) $[C_1, C_2]$ (E) All false

10. At a $\alpha = 0.05$ significance level, the power for this test is, approximately:

(A) 0.93 (B) 0.85 (C) 0.15 (D) 0.07 (E) 0.02

11. Let X be a random variable with probability mas function given by:

$$P_X(x; \theta) = (1 - \theta)^{x-1}\theta, \quad \theta \in (0, 1), \quad x = 1, 2, \cdots$$

We wish to test the null hypothesis $H_0 : \theta = 0.30$ against the alternative hypothesis $H_1 : \theta = 0.60$. In order to do this, a random sample of size $n = 1$ has been taken. The most powerful critical region for this test is of the form:

(A) $[C, \infty)$ (B) $[1, C]$ (C) $[C_1, C_2]^C$ (D) $[C_1, C_2]$ (E) All false