

**STATISTICS APPLIED TO BUSINESS
ADMINISTRATION. ACADEMIC YEAR 2013-2014
SEMINAR 2 (60 MINUTES)**

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EXERCISE 1 (3 POINTS)

Let X be a r.v. with probability density function given by

$$f(x, \theta) = \begin{cases} \frac{2}{(2\theta-1)}(\theta - x) & \text{for } 0 \leq x \leq 1, \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ , a random sample of size n , X_1, X_2, \dots, X_n , is taken, from which the value $a_1 = \bar{x} = 0.35$ is obtained. Find the method of moments **estimator and estimate** of θ , $\hat{\theta}_{MM}$.

EXERCISE 2 (3 POINTS)

Let X be a r.v. with probability mass function given by: $P(X = -1) = P(X = 1) = \theta$, $P(X = 0) = 1 - 2\theta$, $0 < \theta < 1/2$. In order to estimate the parameter θ , a r.s. of size n has been taken, where the value $x = -1$ was obtained n_1 times, the value $x = 0$ was obtained n_2 times, and the value $x = 1$ was obtained n_3 times ($n_1 + n_2 + n_3 = n$).

1. **(1.5 points)** Find the maximum likelihood estimator of θ .
2. **(1.5 points)** Find the method of moments estimator of θ .

EXERCISE 3 (4 POINTS)

Let X_1, X_2, \dots, X_n ($n > 2$) be a r.s. taken from a population that follows a Poisson distribution with parameter λ . Let us consider the following two estimators for the parameter λ :

$$\hat{\lambda}_1 = \frac{X_1 + X_2 + \dots + X_n}{k_1}$$
$$\hat{\lambda}_2 = \frac{3X_1 + X_2 + \dots + X_{n-1} + 3X_n}{k_2}$$

1. **(2 points)** Find the values for the two constants k_1 and k_2 so that the aforementioned estimators, $\hat{\lambda}_1$ and $\hat{\lambda}_2$, are unbiased estimators of λ .
2. **(1 point)** Using the k_1 and k_2 values from the previous item, find out if either one or both of these estimators are consistent. In addition, you should compute the variance for each of these estimators, **providing all relevant details**.
3. **(1 point)** Find out if either one or both of these estimators are efficient. **You should provide all relevant details to justify your answer.**

Remark: This piece of paper should be handed in together with your solutions to the aforementioned exercises. You should also write, both on this piece of paper and in the solutions you provide, the names of the students in your group that have actively participated in this seminar activity.