STATISTICS APPLIED TO BUSINESS ADMINISTRATION
SEMINAR 4

Date: ____________

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EXERCISE 1 (3 POINTS)

Let X be a r.v. with probability density function given by

$$f(x,\theta) = \begin{cases} \frac{x^2}{2\theta^3} e^{-\frac{x}{\theta}} & \text{for } x \geq 0,\ \theta > 0; \\ 0 & \text{otherwise,} \end{cases}$$

where we know that the mean of X is $m = 3\theta$.

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken.

1. **(1.5 points)** Compute the maximum likelihood estimator, $\hat{\theta}_{ML}$, for the parameter θ.
2. **(1.5 points)** Compute the method of moments estimator, $\hat{\theta}_{MM}$, for the parameter θ.

EXERCISE 2 (3 POINTS)

Let X be a r.v. with probability mass function: $P(X = 0) = 2\theta$, $P(X = 1) = 3\theta$, $P(X = 2) = 1 - 5\theta$. In order to estimate the parameter θ, a r.s. has been taken, providing the results: 0, 0, 1, 1, 1, 2, 2, 2, 2, 2.

1. **(1.5 points)** Compute the maximum likelihood estimate of the parameter θ.
2. **(1.5 points)** Compute the method of moments estimate of the parameter θ.

EXERCISE 3 (4 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a $N(m, \sigma^2)$ distribution. Let us consider the following two estimators for the population mean m:

$$\hat{m}_1 = \frac{X_1 + X_2 + \ldots + X_n}{n} = \overline{X}$$

$$\hat{m}_2 = \frac{X_1 + 2X_2 + \ldots + 2X_{n-1} + X_n}{2(n-1)}$$

1. **(1 point)** Find out if either one or both of these estimators are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(2 points)** Find out if either one or both of these estimators are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.
3. **(1 point)** Find out, providing all relevant details, if either one or both of these estimators are efficient. In case you need it, the probability density function for a $N(m, \sigma^2)$ r.v. is given by

$$f_X(x, m, \sigma^2) = \frac{1}{\sqrt{2\pi}\ \sigma}\ e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad -\infty < x < \infty$$

Remark: This piece of paper should be handed in together with your solutions to the aforementioned exercises. You should also write, both on this piece of paper and in the solutions you write, the names of the students in your group that have actively participated in this seminar activity.