STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2024-2025
PRACTICAL EXERCISES 4 AND 5 (40 MINUTES)

Date: ______________

Complete name:____ ID number:______________

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x;\theta) = \begin{cases} (\theta+1)\ 2^{\theta+1}\ x^{\theta} & \text{for } 0 < x < \frac{1}{2}; \\ 0 & \text{otherwise,} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken, and it is known that the mean of the r.v. X is $m = \dfrac{(\theta+1)}{2(\theta+2)}$.

1. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{\mathrm{ML}}$, for the parameter θ.
2. **(5 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{\mathrm{MM}}$, for the parameter θ.

EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a Poisson, $\mathcal{P}(\lambda)$, distribution. Let us consider the following two estimators for the parameter λ:

$$\hat{\lambda}_1 = \frac{X_1 + 3X_2 + \ldots + 3X_{n-1} + 4X_n}{(3n-1)}$$

$$\hat{\lambda}_2 = \frac{2X_1 + X_2 + \ldots + X_{n-1} + 3X_n}{(2n+1)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.