

**STATISTICS APPLIED TO BUSINESS ADMINISTRATION**  
**ACADEMIC YEAR 2023-2024**  
**PRACTICAL EXERCISES 4 AND 5 (40 MINUTES)**

Date: \_\_\_\_\_

Complete name: \_\_\_\_\_ ID number: \_\_\_\_\_

**EXERCISE 1 (10 POINTS)**

Let  $X$  be a r.v. with probability density function given by

$$f(x; \theta) = \begin{cases} (\theta + 1) x^\theta & \text{for } 0 < x < 1, \theta > 0; \\ 0 & \text{otherwise,} \end{cases}$$

In order to estimate the parameter  $\theta$ , a random sample of size  $n$ ,  $X_1, X_2, \dots, X_n$ , is taken.

1. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator,  $\hat{\theta}_{ML}$ , for the parameter  $\theta$ .
2. **(5 points)** Find, providing all relevant details, the method of moments estimator,  $\hat{\theta}_{MM}$ , for the parameter  $\theta$ .

## EXERCISE 2 (10 POINTS)

Let  $X_1, X_2, \dots, X_n$  be a r.s. taken from a population that follows a normal,  $N(\theta, \sigma^2 = 1)$ , distribution. Let us consider the following two estimators for the parameter  $\theta$ :

$$\hat{\theta}_1 = \frac{2X_1 + 3X_2 + \dots + 3X_{n-1} + 3X_n}{(3n - 1)}$$

$$\hat{\theta}_2 = \frac{X_1 + 2X_2 + \dots + 2X_{n-1} + 4X_n}{(3n + 4)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.