

STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2022-2023
PRACTICAL EXERCISES 4 AND 5 (40 MINUTES)

Date: _____

Complete name: _____ ID number: _____

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x; \theta) = \begin{cases} \left(\frac{1}{x\theta}\right) x^{-\frac{1}{\theta}} & \text{for } x \geq 1, \theta > 0; \\ 0 & \text{otherwise,} \end{cases}$$

It is known that the mean of the r.v. X is $E(X) = m_X = (1 - \theta)^{-1}$.

In order to estimate the parameter θ , a random sample of size n , X_1, X_2, \dots, X_n , is taken.

1. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{ML}$, for the parameter θ .
2. **(5 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{MM}$, for the parameter θ .

EXERCISE 2 (10 POINTS)

Let X_1, X_2, \dots, X_n be a r.s. taken from a population that follows a Poisson, $\mathcal{P}(\theta)$, distribution. Let us consider the following two estimators for parameter θ :

$$\hat{\theta}_1 = \frac{X_1 + 4X_2 + \dots + 4X_{n-1} + 3X_n}{4(n-1)}$$

$$\hat{\theta}_2 = \frac{X_1 + 4X_2 + \dots + 4X_{n-1} + X_n}{(n+1)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.