

STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2021-2022
PRACTICAL EXERCISES 4 AND 5 (30 MINUTES)

Date: _____

Complete name: _____ ID number: _____

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x; \theta) = \begin{cases} (\theta + 5) 3^{\theta+5} x^{\theta+4} & \text{for } 0 \leq x \leq \frac{1}{3}; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ , a random sample of size n , X_1, X_2, \dots, X_n , is taken. It is known that the mean of this r.v. is $E(X) = \frac{(\theta+5)}{3(\theta+6)}$.

1. **(5 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{MM}$, for the parameter θ .
2. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{ML}$, for the parameter θ .

EXERCISE 2 (10 POINTS)

Let X_1, X_2, \dots, X_n be a r.s. taken from a population that follows a normal, $N(\theta, \sigma^2 = 4)$, distribution. Let us consider the following two estimators for the mean parameter θ :

$$\hat{\theta}_1 = \frac{4X_1 + 2X_2 + \dots + 2X_{n-1} + 2X_n}{2(n+1)}$$

$$\hat{\theta}_2 = \frac{2X_1 + 3X_2 + \dots + 3X_{n-1} + 2X_n}{(n+1)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.