STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2020-2021
PRACTICAL EXERCISES 4 AND 5 (30 MINUTES)

Date: ____________

Complete name:____ ID number:____________

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x;\theta) = \begin{cases} (\theta+3)\ 4^{\theta+3}\ x^{\theta+2} & \text{for } 0 < x < \frac{1}{4}; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken. It is known that the mean of this r.v. is $\mathrm{E}(X) = \frac{(\theta+3)}{(4\theta+16)}$.

1. **(4 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{\mathrm{MM}}$, for the parameter θ.
2. **(4 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{\mathrm{ML}}$, for the parameter θ.
3. **(2 points)** If a r.s. of size $n = 6$ has been taken, resulting in the sample values 0.18, 0.22, 0.20, 0.24, 0.23 and 0.16, find, providing all relevant details, a maximum likelihood estimate of θ.

EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a normal, $N(\theta, \sigma^2)$, distribution with $\sigma^2 > 0$ known. Let us consider the following two estimators for the mean parameter θ:

$$\hat{\theta}_1 = \frac{3X_1 + 2X_2 + \ldots + 2X_{n-1} + 3X_n}{(2n+2)}$$

$$\hat{\theta}_2 = \frac{5X_1 + 2X_2 + \ldots + 2X_{n-1} + 4X_n}{(2n+2)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.