STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2020-2021
PRACTICAL EXERCISES 4 AND 5 (30 MINUTES)

Date: ______________

Complete name:____ ID number:______________

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x;\theta) = \begin{cases} (\theta+1)\ 2^{\theta+1}\ x^{\theta} & \text{for } 0 < x < \frac{1}{2}; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken. It is known that the mean of this r.v. is $\mathrm{E}(X) = \frac{(\theta+1)}{(2\theta+4)}$.

1. **(4 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{\mathrm{MM}}$, for the parameter θ.
2. **(4 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{\mathrm{ML}}$, for the parameter θ.
3. **(2 points)** If a r.s. of size $n = 5$ has been taken, resulting in the sample values 0.10, 0.15, 0.20, 0.32 and 0.28, find, providing all relevant details, a maximum likelihood estimate of θ.

EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a normal, $N(\theta, \sigma^2)$, distribution with $\sigma^2 > 0$ known. Let us consider the following two estimators for the mean parameter θ:

$$\hat{\theta}_1 = \frac{3X_1 + 2X_2 + \ldots + 2X_{n-1} + 4X_n}{(2n+5)}$$

$$\hat{\theta}_2 = \frac{2X_1 + 4X_2 + \ldots + 4X_{n-1} + 2X_n}{(4n-4)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.
2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.