EXERCISE 1 (10 POINTS)

Let $X$ be a r.v. with probability density function given by

$$f(x; \theta) = \begin{cases} 
(\theta + 1)x^\theta & \text{for } 0 < x < 1, \quad \theta > 0; \\
0 & \text{otherwise}
\end{cases}$$

In order to estimate the parameter $\theta$, a random sample of size $n$, $X_1, X_2, \ldots, X_n$, is taken.

1. \textbf{(5 points)} Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{ML}$, for the parameter $\theta$.

2. \textbf{(5 points)} Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{MM}$, for the parameter $\theta$. 
EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a Poisson, $\mathcal{P}(\lambda)$, distribution. Let us consider the following two estimators for the parameter $\lambda$:

\[
\hat{\lambda}_1 = \frac{X_1 + 2X_2 + \ldots + 2X_{n-1} + X_n}{2n - 2}
\]
\[
\hat{\lambda}_2 = \frac{3X_1 + X_2 + \ldots + X_{n-1} + 3X_n}{(n + 1)}
\]

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.

2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.