STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2018-2019
PRACTICAL EXERCISES 4 AND 5 (30 MINUTES)

Date: ____________

Complete name:____ ID number:___________

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x;\theta) = \begin{cases} (\theta+1)x^{\theta} & \text{for } 0 < x < 1, \ \ \theta > 0; \\ 0 & \text{otherwise} \end{cases}$$

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken.

1. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{\mathrm{ML}}$, for the parameter θ.
2. **(5 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{\mathrm{MM}}$, for the parameter θ.

EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a Poisson, $\mathcal{P}(\lambda)$, distribution. Let us consider the following two estimators for the parameter λ:

$$\hat{\lambda}_1 = \frac{X_1 + 2X_2 + \ldots + 2X_{n-1} + X_n}{2n-2}$$

$$\hat{\lambda}_2 = \frac{3X_1 + X_2 + \ldots + X_{n-1} + 3X_n}{(n+1)}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.

2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.