STATISTICS APPLIED TO BUSINESS ADMINISTRATION
ACADEMIC YEAR 2016-2017
PRACTICAL EXERCISES 4 AND 5 (30 MINUTES)

Date: ____________

Complete name:____ ID number:___________

EXERCISE 1 (10 POINTS)

Let X be a r.v. with probability density function given by

$$f(x;\theta) = \begin{cases} \theta \left(\dfrac{1}{x}\right)^{\theta+1} & \text{for } x \geq 1, \ \ \theta > 1; \\ 0 & \text{otherwise} \end{cases}$$

It is known that the mean of the r.v. X is $m_X = \frac{\theta}{(1-\theta)}$.

In order to estimate the parameter θ, a random sample of size n, $X_1, X_2, \ldots, X_n$, is taken.

1. **(5 points)** Find, providing all relevant details, the maximum likelihood estimator, $\hat{\theta}_{\text{ML}}$, for the parameter θ.
2. **(5 points)** Find, providing all relevant details, the method of moments estimator, $\hat{\theta}_{\text{MM}}$, for the parameter θ.

EXERCISE 2 (10 POINTS)

Let $X_1, X_2, \ldots, X_n$ be a r.s. taken from a population that follows a Poisson, $\mathcal{P}(\theta)$, distribution. Let us consider the following two estimators for the parameter θ:

$$\hat{\theta}_1 = \frac{X_1 + X_2 + \ldots + X_n}{3n}$$

$$\hat{\theta}_2 = \frac{X_1 + 4X_2 + \ldots + 4X_{n-1} + X_n}{4n}$$

1. **(5 points)** Find out if either one or both of these estimators is/are unbiased. In addition, you should compute the bias for each of these estimators.

2. **(5 points)** Find out if either one or both of these estimators is/are consistent. In addition, you should compute the variance for each of these estimators, providing all relevant details.