



# Introducing random heterogeneity in the $\mu$ RRM model

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# Outline

- The  $\mu$ RRM model
- Potential evolutions
- Application
- Conclusions

# Let's first introduce the classical RRM model (Chorus, 2010)

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$$RR_{in} = \sum_{j \neq i} \sum_m \ln(1 + \exp(\beta_m [x_{jmn} - x_{imn}])) + \varepsilon_{in}$$

- $\varepsilon_{in}$  is i.i.d type I EV distributed with variance  $\pi^2/6$
- Choice probabilities correspond to

$$\frac{e^{-R_{in}}}{\sum_j e^{-R_{jn}}}$$

# Now let's move to the $\mu$ RRM model (Cranenburgh *et al.*, 2015)

- This approach generalizes the classical RRM model
- The variance of the error term can be estimated
- The size of the scale parameter corresponds to the profundity of regret imposed by the  $\mu$ RRM model
- « *the notion of profundity of regret refers the extent to which RRM models impose regret minimization behaviour* »

# The $\mu$ RRM model

- $RR_{in} = \sum_{j \neq i} \sum_m \ln(1 + \exp(\frac{\beta_m}{\mu} [x_{jmn} - x_{imn}])) + \varepsilon_{in}$

With  $\varepsilon_{in} \sim i.i.d. EV(0, \mu)$

- Choice probabilities now correspond to

$$\frac{e^{-\mu R_{in}}}{\sum_j e^{-\mu R_{jn}}}$$

# The $\mu$ RRM model – special cases

- When  $\mu$  is arbitrarily large, the  $\mu$ RRM model exhibits linear additive random utility maximization
- When  $\mu$  is arbitrarily small, the difference between the utility one gets from a gain and the regret one gets from a loss is very strong. In this case, the  $\mu$ RRM model takes the form of the P-RRM model
- When  $\mu$  is close to 1 the model corresponds to a normal RRM model

# Why using the the $\mu$ RRM model rather than a Latent class RUM-RRM model?

- The  $\mu$ RRM approach allows to model the profundity of regret in a continuous manner
- It gives a measure of « how much regret there is » rather than « what is the percentage of people expressing a regret minimisation behaviour »
- The  $\mu$ RRM can emulate the results from a LC RUM-RRM while avoiding the estimation issues when  $\mu$  is set up to be random

# Going beyond the $\mu$ RRM model (1)

- In this work, we propose a series of extensions for the  $\mu$ RRM model
- We seek to accommodate heterogeneity in the profundity of regret
- Different people use different decision rules
- Different attributes trigger different choice strategies



# Going beyond the $\mu$ RRM model (2)

- We propose the following extensions:
- The random  $\mu$ RRM model
  - $\mu$  is allowed to be normally distributed across respondents
- The multiple random  $\mu$ RRM model
  - Different, randomly distributed  $\mu$  are estimated for each attribute

# The random $\mu$ RRM model

- $RR_{in} = \sum_{j \neq i} \sum_m \ln(1 + \exp(\frac{\beta_m}{\mu}[x_{jm} - x_{im}])) + \varepsilon_i$
- $\mu$  now corresponds to  $\text{mean}_{\mu} + \text{sd}_{\mu} * \text{random draws}$
- The random draws are normally distributed
- It is a very straightforward change to implement

# The multiple random $\mu$ RRM model

- $RR_{in} = \sum_{j \neq i} \sum_m \mu_m \cdot \ln(1 + \exp(\frac{\beta_m}{\mu_m}[x_{jm} - x_{im}])) + \varepsilon_i$
- Each  $\mu_m$  now corresponds to  $\text{mean}_{\mu_m} + \text{sd}_{\mu_m} * \text{random draws}$
- This model does not seem to converge well unless we estimate a full variance\_covariance matrix for the random draws
- In this case, the choice probability correspond to:

$$\frac{e^{-R_{in}}}{\sum_j e^{-R_{jn}}}$$

# Application

- Our dataset comes from an Australian regional mobility survey. Each respondent faced 10 choice tasks involving a choice between four labelled alternatives: plane and taxi, plane and shuttle, car, coach and taxi
- Attributes:
  - departure time
  - average travel time
  - travel time early
  - travel time late
  - Cost
  - wait time for transfer service
  - cost of transfer service
  - Duration for transfer service
- 811 respondents

	uRRM		Random uRRM		Multiple random uRRM	
	est	t ratio	est	t ratio	est	t ratio
bdepatime	<b>1.74</b>	<b>9.44</b>	<b>1.65</b>	<b>5.21</b>	<b>1.95</b>	<b>8.49</b>
btravtime	<b>-0.76</b>	<b>-11.51</b>	<b>-0.81</b>	<b>-4.48</b>	<b>-0.84</b>	<b>-3.01</b>
bearlymin	-2.47	-1.55	-2.56	-1.78	<b>-2.99</b>	<b>-3.03</b>
blatemin	<b>-1.41</b>	<b>-8.33</b>	<b>-1.44</b>	<b>-4.45</b>	-1.45	-1.61
btravcost	<b>-2.06</b>	<b>-11.70</b>	<b>-2.04</b>	<b>-10.45</b>	<b>-2.02</b>	<b>-13.16</b>
bwaittime	<b>-2.52</b>	<b>-3.99</b>	<b>-2.45</b>	<b>-3.99</b>	<b>-2.73</b>	<b>-3.27</b>
btrantime	<b>6.36</b>	<b>2.20</b>	<b>5.25</b>	<b>4.12</b>	<b>4.95</b>	<b>3.99</b>
btrancost	<b>-3.55</b>	<b>3.11</b>	<b>-3.01</b>	<b>2.54</b>	<b>-2.86</b>	<b>-2.05</b>
alt1	<b>-0.16</b>	<b>-7.89</b>	<b>-0.19</b>	<b>-5.58</b>	<b>-0.44</b>	<b>-2.51</b>
alt2	-0.65	-1.47	-0.48	-1.42	0.19	1.12
alt3	<b>-0.52</b>	<b>-6.47</b>	<b>-0.52</b>	<b>-5.54</b>	<b>-1.75</b>	<b>-22.25</b>

	uRRM		Random uRRM		Multiple random uRRM	
	est	t ratio	est	t ratio	est	t ratio
mu1	<b>1.21</b>	<b>3.92</b>	<b>1.14</b>	<b>3.87</b>	<b>3.10</b>	<b>3.15</b>
mu2					<b>23.38</b>	<b>3.80</b>
mu3					<b>-0.34</b>	<b>-2.89</b>
mu4					-0.65	-1.49
mu5					6.27	0.01
mu6					<b>-0.05</b>	<b>-2.56</b>
mu7					<b>-0.16</b>	<b>-2.25</b>
mu8					<b>-0.26</b>	<b>-2.00</b>
sdmu1			0.08	0.78	-0.05	0.01
sdmu2					0.11	<b>2.87</b>
sdmu3					0.34	<b>1.99</b>
sdmu4					-0.43	-1.36
sdmu5					0.14	-0.93
sdmu6					<b>0.14</b>	<b>2.24</b>
sdmu7					0.26	0.89
sdmu8					<b>0.15</b>	<b>1.94</b>
AIC	17963.13		179851.54		17880.81	
LL	-8969.565		-8958.781		-8885.404	

# Discussion

- First results look promising:
  - Significant observed heterogeneity in the profundity of regret
  - Significant random heterogeneity
- Model performed (much) better than a LC RUM RRM
  - More convenient way to introduce heterogeneity in decision rules in SP survey
- Some challenges: « more convenient » doesn't mean perfect (lots of issues with local optima)

ANY  
QUESTIONS  
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