EXAMINATION - INTRODUCTION TO ECONOMETRICS (LADE), 3rd YEAR
June 21, 2006 Time: 2h 30m.

SURNAME 1:_____ **SURNAME 2:**_____
NAME:_____ **DNI:**_____ **GROUP:**_____

EXAMINATION ATTEMPT: [1] [2] [3] [4] [5] [6] [7] (Please tick the appropriate box)
Did you do the test? [YES] [NO] (Please tick the appropriate box)

QUESTION 1 (marks)

We want to analyse telephone bill expenditure as a function of family income and family size. In order to do that, we propose the following model:

$$Y_i = \beta_1 + \beta_2\, I_i + \beta_3\, S_i + u_i \qquad i = 1, \ldots, 15 \qquad u_i \sim NID(0, \sigma^2) \tag{1}$$

where:

- Y_i: annual expenditure on fixed telephone by a family unit, in Euros.
- I_i: annual family income, in thousands of Euros.
- S_i: number of members of the family unit.

The data set obtained from 15 families includes the following information:

$$(X'X)^{-1} = \begin{pmatrix} 0.2782 & -0.00409 & -0.03859 \\ -0.00409 & 0.0002918 & -0.001955 \\ -0.03859 & -0.001955 & 0.04139 \end{pmatrix}$$

$$\begin{array}{lll} \sum Y_i = 5475 & \sum Y_i I_i = 195375 & \sum Y_i^2 = 2340325 \\ \sum I_i = 445 & \sum Y_i S_i = 15670 & \sum I_i^2 = 18213 \\ \sum S_i = 35 & \sum I_i S_i = 1275 & \sum S_i^2 = 117 \end{array}$$

A. Estimate the proposed model by the Ordinary Least Squares method. Give an interpretation of the estimated coefficients. Do they have reasonable signs?

B. Calculate and give an interpretation of a goodness-of-fit measure (for the obtained fit).

C. Obtain a theoretical expression for the variance-covariance matrix of the OLS estimator of model (1). Write down the basic assumptions needed for the demonstration.

D. Estimate the variance of the disturbances.

E. Are the explanatory variables individually significant? and jointly? Carry out the suitable testing procedures.

F. Test whether the effect on average fixed telephone annual expenditure of a unit change in family size is 10 times bigger than the effect of a unit change in annual family income.

G. Is it possible that a family of 2 members that has an annual income of 30.000 Euros may have an annual telephone expenditure of 700 Euros? Justify your answer.

We think that the availability of mobile telephones may affect the fixed telephone bill. Therefore, we also carried out the following regression:

$$\underset{\text{(t-estat.)}}{\widehat{Y}_i} = \underset{(2.95)}{110.372} + \underset{(3.52)}{4.095}\, I_i + \underset{(2.41)}{46.642}\, S_i + \underset{(0,.4)}{17.36}\, M_i \qquad R^2 = 0.883 \qquad (2)$$

where M_i is the number of mobile telephones available in the family unit.

H. Is the new variable M_i significant? What are the properties of the OLS estimators in model (2)? Taking all your answers into account, what model would you choose to explain expenditure on fixed telephone?

QUESTION 2 (marks)

We have data on 32 car models regarding their petrol consumption, C_i, (litres per 100 km.), weight, W_i, (in Tons) and whether they have 4, 6 or 8 cylinders. In order to take into account this last feature, we have defined the following dummy variables:

$$D4_i = \begin{cases} 1 & \text{if } i \text{ is a car with 4 cylinders} \\ 0 & \text{if } i \text{ is a car with 6 or 8 cylinders} \end{cases}$$

$$M4_i = \begin{cases} 1 & \text{if } i \text{ is a car with 6 or 8 cylinders} \\ 0 & \text{if } i \text{ is a car with 4 cylinders} \end{cases}$$

$$D6_i = \begin{cases} 1 & \text{if } i \text{ is a car with 6 cylinders} \\ 0 & \text{if } i \text{ is a car with 4 or 8 cylinders} \end{cases}$$

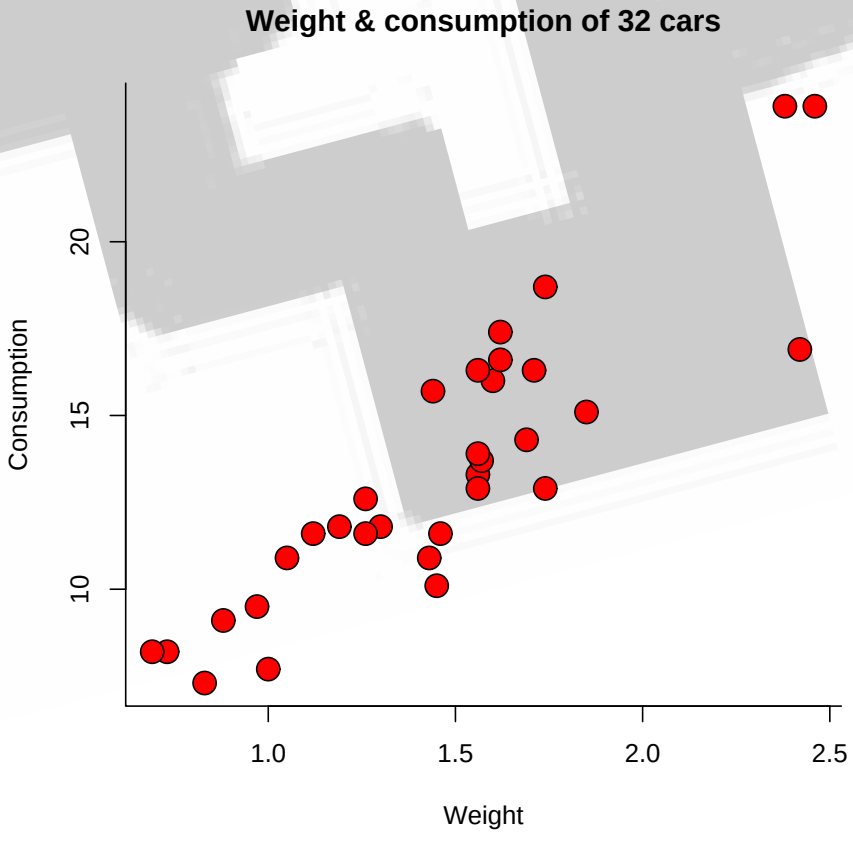
$$D8_i = \begin{cases} 1 & \text{if } i \text{ is a car with 8 cylinders} \\ 0 & \text{if } i \text{ is a car with 4 or 6 cylinders} \end{cases}$$

Initially, we estimated the following two models:

$$\underset{\text{(t-estat.)}}{\widehat{C}_i} = \underset{(1.29)}{1.51} + \underset{(10.63)}{8.19}\ W_i \qquad \sum \hat{u}_i^2 = 118.3948 \tag{3}$$

$$\underset{\text{(t-estat.)}}{\widehat{C}_i} = \underset{(1.71)}{2.12}\ D4_i + \underset{(1.89)}{3.46}\ M4_i + \underset{(6.74)}{7.17}\ W_i \qquad \sum \hat{u}_i^2 = 101.7830 \tag{4}$$

A. Draw the sample regression lines (3) and (4) within the following data points:



B. In relation to model (4), write down the statistic for testing that the number of cylinders doesn't affect fuel consumption. Carry out the test with the available information.

Consider now the following model:

$$C_i = \alpha_1 D4_i + \alpha_2 D6_i + \alpha_3 D8_i + \beta W_i + u_i \quad i = 1, \ldots, 32 \tag{5}$$

C. Give an interpretation of the coefficients α_2 and α_3.

OLS estimation results for model (5) are:

$$\underset{\text{(t-estat.)}}{\widehat{C}_i} = \underset{(2.53)}{3.33}\, D4_i + \underset{(2.33)}{4.13}\, D6_i + \underset{(2.82)}{6.06}\, D8_i + \underset{(5.19)}{6.00}\, W_i \qquad \sum \hat{u}_i^2 = 88.44 \tag{6}$$

D. Given these results, which of the three specifications is better? Justify your answer.

E. Explain the differences between models (4) and (5).

F. For the model chosen in section D, in how much do you estimate the average fuel consumption per 100 km. of a 4-cylinder car? And of an 8-cylinder car? And if both weigh 1.4 Tons?