EXAMINATION - INTRODUCTION TO ECONOMETRICS (LADE), 3rd YEAR
February 8, 2006 Time: 2h 30m.

SURNAME 1:_____ **SURNAME 2:**_____
NAME:_____ **DNI:**_____ **GROUP:**_____

EXAMINATION ATTEMPT: [1] [2] [3] [4] [5] [6] [7] (Please tick the appropriate box)

QUESTION 1 (10 marks)

A student of Management wants to analyse the **fuel** consumption of his car. To do this, he proposes the following model:

$$F_t = \alpha + \beta_1 M_t + \beta_2 C_t + u_t \qquad t = 1, \ldots, T \qquad u_t \sim N(0, \sigma^2) \tag{1}$$

where:

F_t is the number of litres of **fuel** consumed during week t.

M_t are kilometres driven on **motorways** during week t, in hundreds of kilometres.

C_t are kilometres driven on **city** streets and roads during week t, in hundreds of kilometres.

1. Describe the Ordinary Least Square (OLS) criteria in this case and use it in this model to obtain the Normal Equations.

2. One of the properties of the Sample Regression Function is that the arithmetic mean of the regression residuals is zero. Is this fulfilled in this case? Explain the result using the Normal Equations obtained in the previous section.

He (the student) has collected the first seven weekly observations after the purchase of the car. The data gathered can be summed up as follows:

t	F_t	M_t	C_t
1	49	3	3
2	40	5.4	0
3	43	4.4	1
4	38	5.4	0
5	42	4.4	1
6	46	3.6	2
7	43	6	0

$$\sum M_t = 32.2 \quad \sum C_t = 7 \quad \sum F_t = 301$$
$$\sum M_t^2 = 155 \quad \sum C_t^2 = 15 \quad \sum F_t^2 = 13023$$
$$\sum M_t C_t = 25 \quad \sum F_t M_t = 1365.8 \quad \sum F_t C_t = 324$$

$$(X'X)^{-1} = \begin{pmatrix} 75.89286 & -13.75 & -12.50 \\ -13.75000 & 2.50 & 2.25 \\ -12.50000 & 2.25 & 2.15 \end{pmatrix}$$

3. Calculate the following values:

   - How many litres of fuel did the car consume in the third week after its purchase?
   - How many kilometres has it been driven in the city since its purchase?
   - What is the **sample mean** of kilometres driven on motorways?

Results of the least squared estimation of model (1) are as follows:

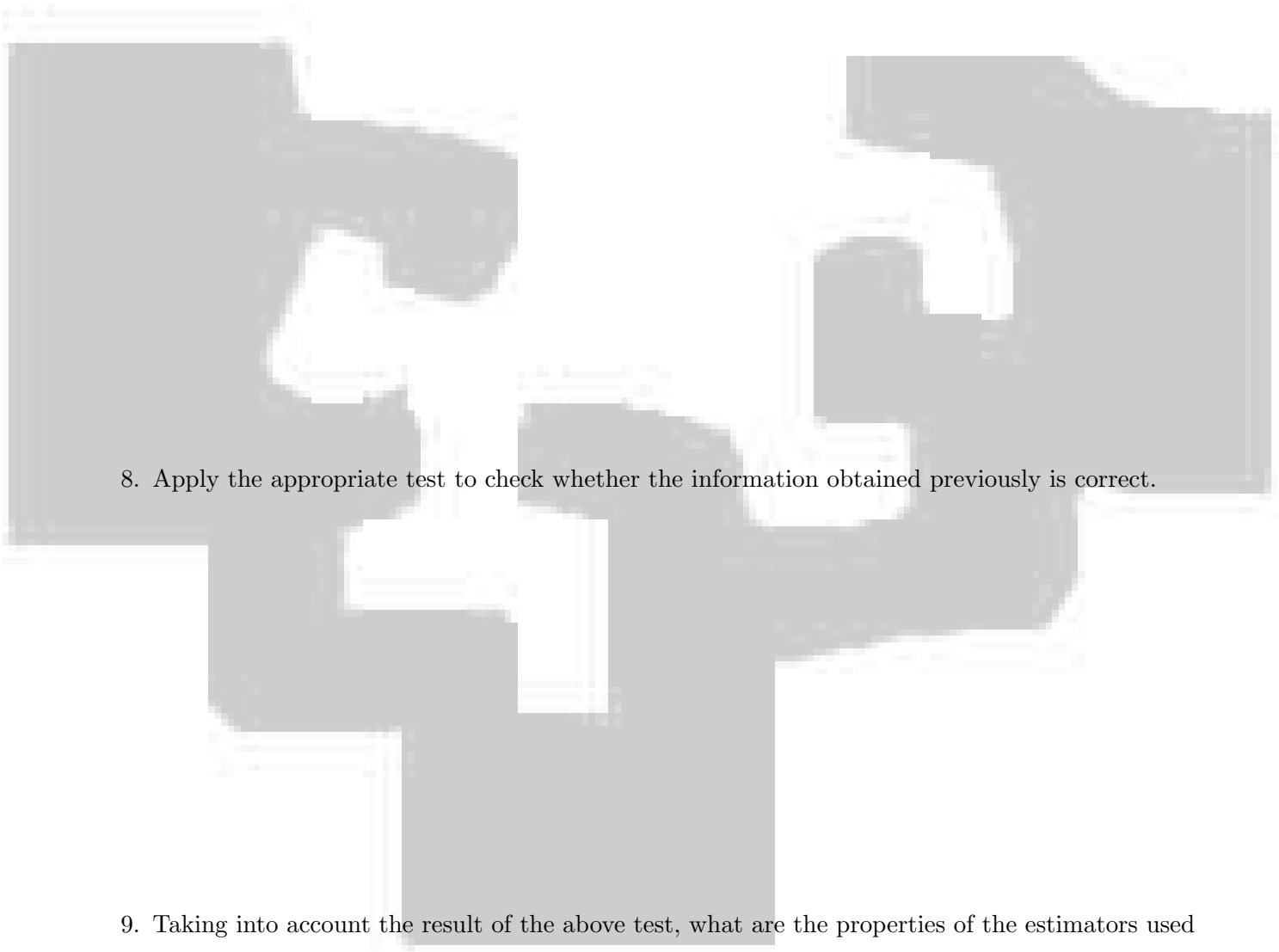
$$\widehat{G}_t = 14 + 4.75M_t + 7.15C_t \qquad t = 1, \ldots, 7 \qquad R^2 = 0.93937$$

4. What is the OLS residual on estimating the litres of fuel consumed during the last week?

5. What is the estimated average fuel consumption per hundred kilometres when driving **only** in the city? And if **only** on motorways?

6. Estimate the unbiased variance and covariance matrix of the OLS estimators.

7. Suppose that, on driving an additional 100 kilometres, the average fuel consumption doubles if driving in the city instead of on motorways. **Estimate** model (1) taking into account this information and calculate the Residual Sum of Squares.



8. Apply the appropriate test to check whether the information obtained previously is correct.

9. Taking into account the result of the above test, what are the properties of the estimators used to estimate the model in section 7?

QUESTION 2 (7 marks)

In order to analyse the salaries of a group of employees of a firm we have data of 60 employees as to their salary in thousands of euros (Y), work experience in years (X) and gender (S). We have estimated the following relationships:

$$\underset{\text{(t-estad.)}}{\widehat{Y}_i} = \underset{(12.17)}{1.7618}\, S1_i + \underset{(11.52)}{1.8556}\, S2_i + \underset{(9.16)}{0.0701}\, X_i \qquad (2)$$

$$R^2 = 0.8644 \qquad \sum_i \widehat{u}_i^2 = 1.1725$$

$$\underset{\text{(t-estad.)}}{\widehat{Y}_i} = \underset{(11.52)}{1.8556} \underset{(-0.65)}{-0.0937}\, S1_i + \underset{(9.16)}{0.0701}\, X_i \qquad (3)$$

$$R^2 = 0.8644 \qquad \sum_i \widehat{u}_i^2 = 1.1725$$

where $S1_i$ is a dummy variable that takes the value of 1 if the person is male and zero otherwise. Conversely, the variable $S2_i$ takes the value 1 if the person is female and zero otherwise.

1. Why do the Coefficient of Determination and the Residual Sum of Squares coincide in both cases? Give reasons for your answer.

2. Test the **individual** significance of both the gender variable **and** the experience variable.

Later, and in view of the results obtained, we consider the following estimations:

$$\underset{\text{(t-estad.)}}{\widehat{Y}_i} = \underset{(14.06)}{1.7932} + \underset{(9.60)}{0.0709}\, X_i \qquad (4)$$

$$R^2 = 0.8602 \qquad \sum_i \widehat{u}_i^2 = 1.2086$$

$$\underset{\text{(t-estad.)}}{\widehat{Y}_i} = \underset{(10.39)}{2.2499} \underset{(-2.43)}{-0.4882}\, ES_i + \underset{(5.01)}{0.05141}\, X_i \qquad (5)$$

$$R^2 = 0.9018 \qquad \sum_i \widehat{u}_i^2 = 0.8487$$

$$\underset{\text{(t-estad.)}}{\widehat{Y}_i} = \underset{(9.82)}{2.2808} + \underset{(4.85)}{0.05129}\,X_i \underset{(-0.49)}{-0.06227}\,S1_i \underset{(-2.30)}{-0.4770}\,ES_i \qquad (6)$$

$$R^2 = 0.9037 \qquad \sum_i \widehat{u}_i^2 = 0.8329$$

where ES_i is a dummy variable that takes the value 1 if the person has primary or secondary level studies and 0 if they have higher studies.

3. Based on the results of model (6), what is the estimated average salary of a male with secondary school education and no work experience?

4. Test the **joint** significance of the education and gender variables together.

5. Based on the information available so far, which specification is more accurate? Why?

6. In your opinion, if the sample available consisted of 60 employees all with higher studies, would there be any problem in estimating the effects of the experience, gender and education variables on salary?