A bootstrap approximation for the distribution of the Local Whittle estimator

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Introduction

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The bootstrap has to deal with the strong dependence and lack of mixing conditions in long memory series $\Rightarrow$ traditional tools not valid.

Long memory

Long memory series x_t with spectral density

$$f(\lambda) = |\lambda|^{-2d} g(\lambda) \quad \lambda \in [-\pi, \pi]$$

- d is the memory parameter (to be estimated):
  - $d \in (-0.5, 0.5)$ guarantees stationarity and invertibility.
  - $d \geq 0.5$: $f(\lambda)$ is a pseudo-spectral density function.
- $g(\lambda)$ is any function positive and bounded over $\lambda \in [-\pi, \pi]$ satisfying

$$g(\lambda) = g(0) + \Delta(\lambda) \,, \quad |\Delta(\lambda)| \leq C_1 |\lambda|^{\alpha}$$

for constant C_1 and local spectral smoothness parameter $\alpha > 0$ ($\alpha = 2$ in ARFIMA models).

Local Whittle estimation

- The LW estimate $\hat{d}$ is obtained by minimizing

$$R(d) = \log\left(\frac{1}{m}\sum_{j=1}^{m}\lambda_j^{2d} I_j\right) - \frac{2d}{m}\sum_{j=1}^{m}\log\lambda_j$$

where I_j is the periodogram of x_t, $t = 1, 2, ..., n$, at Fourier frequency $\lambda_j = 2\pi j/n$

$$I_j = I(\lambda_j) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} x_t \exp(-i\lambda_j t)\right|^2$$

and m is the bandwidth that represents the number of frequencies used in the estimation.

Asymptotic Properties

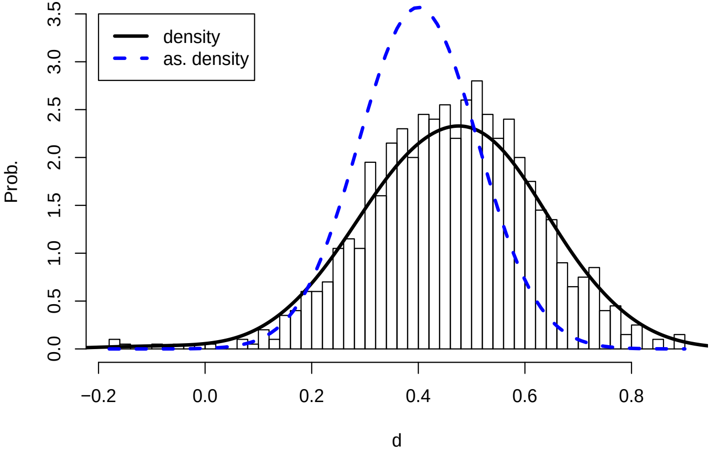
- $\hat{d} \xrightarrow{p} d$ for $-1 < d \leq 1$.
- $\hat{d} \xrightarrow{p} 1$ for $d > 1$.
- $\sqrt{m}(\hat{d} - d) \xrightarrow{d} \frac{1}{2}U_1$ for $-1/2 < d < 3/4$.
- $\sqrt{m}(\hat{d} - d) \xrightarrow{d} \frac{1}{2}U_1 + J(d)U_2^2$ for $d = \frac{3}{4}$.
- $m^{2-2d}(\hat{d} - d) \xrightarrow{d} J(d)U_2^2$ for $d \in (3/4, 1)$.
- $\sqrt{m}(\hat{d} - d) \xrightarrow{d} \frac{-U_1+\sqrt{2}U_2U_3}{2(1+U_3^2)}$ for $d = 1$

U_i, $i = 1, 2, 3$, mutually independent standard normal r.v.'s and $J(d)$ is a function of d different for type I and II long memory.

Asymptotic vs exact distribution ($-1/2 < d < 3/4$)

- Problem: Poor approximation in finite samples.

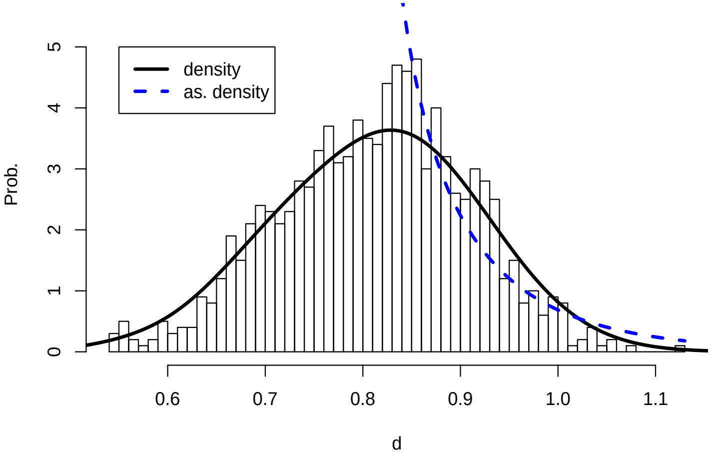
Figure : LW pdf, ARFIMA(1,0.4,0), $\phi = 0.3$, $n = 128$, $m = 20$



Asymptotic vs exact distribution ($3/4 < d < 1$)

- Problem: Poor approximation even in large samples

Figure : LWE pdf, ARFIMA(0,0.8,0),$n = 512$, $m = 40$



Unknown asymptotic distribution

- Problem: LW is consistent but the asymptotic distribution is unknown:
  - Non invertible ARFIMA ($d < -1/2$), consistency shown in Shimotsu and Phillips (2006).
  - Non linear transformations of long memory series, consistency shown in Dalla et al. (2005).

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In all these situations bootstrap can be a useful tool to approximate distributional characteristics of the LW estimator.

Bootstrap approximation

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• I_j shows a marked structure and ordinates are not asymptotically independent at frequencies close to 0 ⇒ resample the standardize periodogram $I_j/f(\lambda_j)$ (Franke and Härdle, 1992 and Dahlhaus and Janas, 1996).

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• Consistent estimation of $f(\lambda_j)$ is not trivial (Kim and Nordman, 2013), especially at frequencies close to zero where traditional (kernel based) estimators are not consistent (Velasco, 2003).

Bootstrap approximation

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- Two options to standardize I_j:

1. Use the estimator proposed by Hidalgo and Yajima (2002) whose consistency at every Fourier frequency has been recently established in Arteche (2013),

$$\hat{f}_j = \hat{f}(\lambda_j) = \frac{|\lambda_j|^{-2\hat{d}}}{2m^* + \mathbf{1}_{j>m^*}} \sum_{k=-m^*, \neq -v}^{m^*} |\lambda_j + \lambda_k|^{2\hat{d}} I(\lambda_j + \lambda_k)$$

for $\lambda_j = 2\pi j/n$, $j = 1, ..., [n/2]$, $\hat{d}$ the LW estimator. Resample the *Studentized periodogram* $\hat{v}_j^{(0)} = I_j/\hat{f}_j$ as if they were i.i.d.

Bootstrap approximation

2. Standardize I_j with $\lambda_j^{-2\hat{d}}$ and resample the *locally standardized periodogram* $\hat{v}_j^{(1)} = I_j \lambda_j^{2\hat{d}}$. Some structure remains (no i.i.d.) $\Rightarrow$ local bootstrap (Paparoditis and Politis, 1999) to maintain the structure in the bootstrap samples.

Frequency domain bootstrap: Steps

1. Obtain $\hat{v}_j^{(i)}$, $i = 0, 1$, for $j = 1, ..., [n/2]$ with a bandwidth m for $\hat{d}$, and m^* for $\hat{f}_j$.

2. Let $k_n = [n/2]$ for $\hat{v}_j^{(0)}$ and select a *resampling width* $k_n \in \mathcal{N}$, $k_n \leq [n/2]$ for $\hat{v}_j^{(1)}$.

3. Define i.i.d. discrete random variables $S_1, ..., S_m$ taking values in the set $\{0, \pm 1, , ..., \pm k_n\}$ with equal probability $1/(2k_n + 1)$.

4. Generate B bootstrap series $\hat{v}_{bj}^{*(i)} = \hat{v}_{|j+S_j|}^{(i)}$ if $|j + S_j| > 0$, $\hat{v}_{bj}^{*(i)} = \hat{v}_1^{(i)}$ if $j + S_j = 0$ for $b = 1, 2, ..., B$ and $j = 1, ..., m$.

5. Generate B bootstrap samples for the periodogram $I_{bj}^{*(1)} = \lambda_j^{-2\hat{d}} \hat{v}_{bj}^{*(1)}$, $I_{bj}^{*(0)} = \hat{f}_j \hat{v}_{bj}^{*(0)}$ for $b = 1, 2, ..., B$.

6. Obtain the B bootstrap LW estimates $\hat{d}_b^{*(i)}$, $b = 1, ..., B$ by minimizing $R(d)$ with the periodogram I_j replaced by $I_{bj}^{*(i)}$.

Frequency domain bootstrap: Remarks

Remark 1: m remains fixed. The procedure is designed to obtain bootstrap replicates of d for a given m.

Remark 2: The user has to select m^* or k_n:

- k_n based on the form of $\hat{v}_j^{(1)}$, the higher the structure the lower k_n should be chosen to keep the global structure of $\hat{v}_j^{(1)}$ in the bootstrap samples.
- m^* can be chosen similarly because $\hat{f}_j$ is based on a moving average of neighbour $\hat{v}_k^{(1)}$s.

Monte Carlo: Stationary series

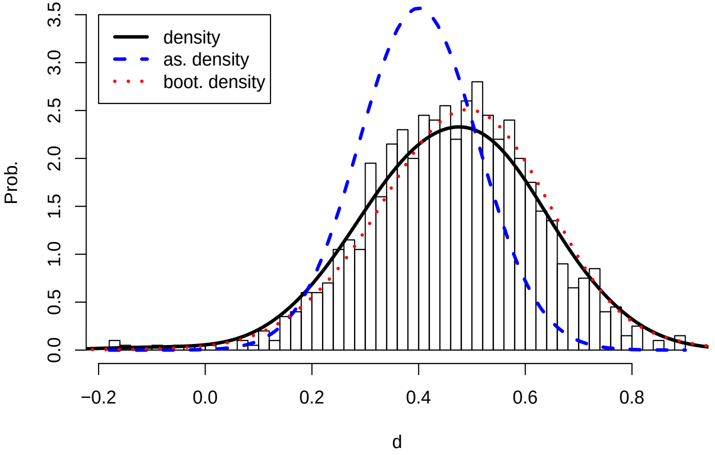
$$(1-\phi L)(1-L)^d X_t = \varepsilon_t, \quad t = 1, 2, ..., n,$$

- $n = 128$.
- $\varepsilon_t \sim NID(0,1)$.
- $d = 0,\ 0.4$.
- $\phi = 0.3,\ 0.8$.
- $m = 5, 10, 20$.
- $m^* = 3, 5, 7$.
- $k_n = 2, 5, 10, 20$.
- $B = 999$ bootstrap replications.
- 1000 simulations.

Stationary case

- Asymptotic distribution vs exact distribution ($-1/2 < d < 3/4$).

Figure : LWE pdf, ARFIMA(1,0.4,0), $\phi = 0.3$, $n = 128$, $m = 20$



Some existing improvements

- *Variance improvement*: Use the Hessian based approximation:

$$\widehat{var}(\hat{d}) = \left(4 \sum_{j=1}^{m} \left(\log \lambda_j - \frac{1}{m} \sum_{k=1}^{m} \log \lambda_k \right)^2 \right)^{-1},$$

instead of the asymptotic variance $1/4m$ (Hurvich and Chen, 2000 and Arteche, 2006).

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instead of the asymptotic variance $1/4m$ (Hurvich and Chen, 2000 and Arteche, 2006).

• *Bias improvement*: Use Edgeworth expansions (Giraitis and Robinson, 2003)

$$\sup_{y\in R}\left|P(2\sqrt{m}(\hat{d}-d)\leq y) - \Phi(y) - \phi(y)\theta_1\sqrt{m}\frac{m^2}{n^2}\right| = o\left(\sqrt{m}\frac{m^2}{n^2}\right)$$

Confidence Intervals

- Option 1. The asymptotic distribution

$$CI^1_{1-\alpha} = \left(\hat{d} - 0.5m^{-1/2} z_{1-\frac{\alpha}{2}}; \quad \hat{d} - 0.5m^{-1/2} z_{\frac{\alpha}{2}}\right)$$

z_α is the $100 \cdot \alpha$th percentile of the as. distribution ($N(0,1)$).

- Option 2. Using Hessian based approximation of the variance

$$CI^2_{1-\alpha} = \left(\hat{d} - \sqrt{\hat{var}(\hat{d})} z_{1-\frac{\alpha}{2}}; \quad \hat{d} - \sqrt{\hat{var}(\hat{d})} z_{\frac{\alpha}{2}}\right).$$

- Option 3. Using Giraitis and Robinson (2003) proposal,

$$CI^3_{1-\alpha}(\hat{\theta}_1) = \left(\hat{d} + \frac{\hat{\theta}_1}{2}\frac{m^2}{n^2} - 0.5m^{-1/2} z_{1-\frac{\alpha}{2}}; \hat{d} + \frac{\hat{\theta}_1}{2}\frac{m^2}{n^2} - 0.5m^{-1/2} z_{\frac{\alpha}{2}}\right)$$

Confidence Intervals

- Option $4(m^*)$. Using the global bootstrap strategy based on the *Studentized periodogram* $\hat{v}_j^{(0)}$ for different m^*.

$$CI^4_{(1-\alpha)}(m^*) = \left(\hat{d}^{*(0)}_{\left((B+1)\left(\frac{\alpha}{2}\right)\right)} \quad ; \quad \hat{d}^{*(0)}_{\left((B+1)\left(1-\frac{\alpha}{2}\right)\right)} \right),$$

where $\hat{d}^{*(0)}_{(j)}$ denotes the jth ordered value of the bootstrap estimates of d.

- Option $5(k_n)$. $CI^5_{(1-\alpha)}(k_n)$ is similarly calculated but using the local bootstrap strategy based on the *locally standardized periodogram* $\hat{v}_j^{(1)}$ for different resampling widths k_n.

95% Confidence Intervals

Table : Coverage for an $ARFIMA(1, d, 0)$ with $\phi = 0.3$ ($n = 128$)

	d=0			d=0.4		
	m=5	m=10	m=20	m=5	m=10	m=20
$CI^1_{0.95}$	0.662	0.811	0.823	0.678	0.795	0.815
	(0.8779)	(0.620)	(0.438)	(0.877)	(0.620)	(0.438)
$CI^2_{0.95}$	0.894	0.925	0.912	0.887	0.917	0.905
	(1.542)	(0.891)	(0.553)	(1.542)	(0.891)	(0.553)
$CI^3_{0.95}(\hat{\theta}_1)$	0.662	0.811	0.823	0.678	0.795	0.815
	(0.877)	(0.620)	(0.438)	(0.877)	(0.620)	(0.438)
$CI^4_{0.95}(3)$	0.940	0.936	0.932	0.942	0.917	0.926
	(1.713)	(0.935)	(0.549)	(1.750)	(0.929)	(0.546)
$CI^4_{0.95}(5)$	0.967	0.950	0.937	0.960	0.948	0.943
	(1.771)	(0.963)	(0.564)	(1.800)	(0.958)	(0.561)
$CI^4_{0.95}(7)$	0.962	0.969	0.953	0.957	0.965	0.948
	(1.787)	(0.977)	(0.574)	(1.817)	(0.970)	(0.570)

The top number in each cell is the coverage frequency. The bottom number (in round brackets) is the length of the interval.

95% Confidence Intervals

Table : Coverage for an $ARFIMA(1, d, 0)$ with $\phi = 0.3$ ($n = 128$)

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	(1.787)	(0.977)	(0.574)	(1.817)	(0.970)	(0.570)
$CI^5_{0.95}(2)$	0.847	0.873	0.852	0.846	0.841	0.836
	(1.316)	(0.804)	(0.501)	(1.327)	(0.802)	(0.501)
$CI^5_{0.95}(5)$	0.946	0.924	0.904	0.947	0.929	0.899
	(1.543)	(0.891)	(0.546)	(1.564)	(0.894)	(0.546)
$CI^5_{0.95}(10)$	0.982	0.964	0.941	0.977	0.956	0.934
	(1.775)	(0.929)	(0.562)	(1.807)	(0.934)	(0.562)
$CI^5_{0.95}(20)$	1.000	0.975	0.957	1.000	0.966	0.958
	(1.958)	(1.013)	(0.573)	(2.001)	(1.013)	(0.575)

The top number in each cell is the coverage frequency. The bottom number (in round brackets) is the length of the interval.

95% Confidence Intervals

Table : Coverage for an $ARFIMA(1, d, 0)$ with $\phi = 0.8$ $(n = 128)$

	d=0			d=0.4		
	m=5	m=10	m=20	m=5	m=10	m=20
$CI^1_{0.95}$	0.627	0.402	0.022	0.606	0.419	0.021
	(0.877)	(0.620)	(0.438)	(0.876)	(0.620)	(0.438)
$CI^2_{0.95}$	0.900	0.623	0.043	0.873	0.641	0.057
	(1.542)	(0.891)	(0.553)	(1.542)	(0.891)	(0.553)
$CI^3_{0.95}(\hat{\theta}_1)$	0.626	0.418	0.033	0.604	0.433	0.040
	(0.877)	(0.620)	(0.438)	(0.877)	(0.620)	(0.438)
$CI^4_{0.95}(3)$	0.977	0.764	0.068	0.965	0.775	0.076
	(1.751)	(0.935)	(0.557)	(1.722)	(0.912)	(0.543)
$CI^4_{0.95}(5)$	0.981	0.772	0.058	0.979	0.775	0.072
	(1.809)	(0.965)	(0.572)	(1.766)	(0.940)	(0.556)
$CI^4_{0.95}(7)$	0.975	0.762	0.051	0.968	0.757	0.065
	(1.828)	(0.978)	(0.580)	(1.785)	(0.953)	(0.564)

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	(1.828)	(0.978)	(0.580)	(1.785)	(0.953)	(0.564)
$CI^5_{0.95}(2)$	0.788	0.557	0.067	0.797	0.550	0.091
	(1.323)	(0.835)	(0.545)	(1.321)	(0.827)	(0.536)
$CI^5_{0.95}(5)$	0.906	0.646	0.081	0.903	0.628	0.100
	(1.581)	(0.927)	(0.591)	(1.574)	(0.915)	(0.580)
$CI^5_{0.95}(10)$	0.951	0.666	0.053	0.953	0.671	0.069
	(1.876)	(0.972)	(0.592)	(1.863)	(0.957)	(0.581)
$CI^5_{0.95}(20)$	0.977	0.749	0.046	0.981	0.746	0.056
	(2.193)	(1.088)	(0.602)	(2.163)	(1.063)	(0.590)

The top number in each cell is the coverage frequency. The bottom number (in round brackets) is the length of the interval.

Skewness and Kurtosis

Table : Skewness and Kurtosis, $ARFIMA(1, 0.4, 0)$, $(n = 128)$

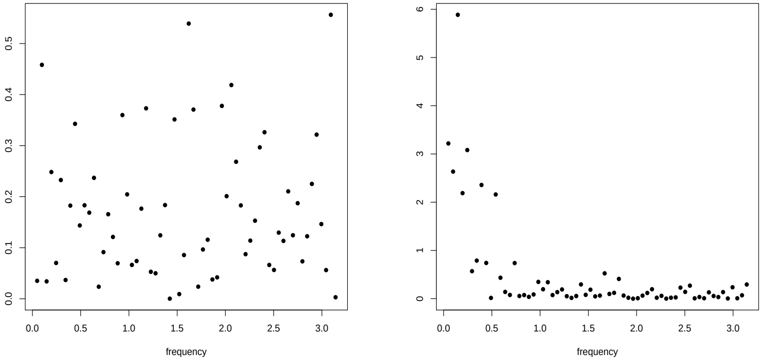
	$\phi = 0.3$			$\phi = 0.8$		
	m=5	m=10	m=20	m=5	m=10	m=20
Monte Carlo	-0.222	-0.351	-0.285	-0.409	-0.356	-0.179
	3.524	3.736	3.572	3.618	3.938	3.807
Option 4(m^*=3)	-0.280	-0.356	-0.282	-0.370	-0.372	-0.235
	3.628	3.696	3.472	3.773	3.776	3.458
Option 4(m^*=5)	-0.262	-0.329	-0.294	-0.356	-0.349	-0.282
	3.587	3.639	3.429	3.705	3.699	3.437
Option 4(m^*=7)	-0.258	-0.304	-0.291	-0.352	-0.313	-0.291
	3.558	3.609	3.416	3.674	3.653	3.423
Option 5(k_n=2)	0.215	0.045	-0.103	0.202	0.046	-0.098
	3.087	3.458	3.409	3.013	3.337	3.300
Option 5(k_n=5)	-0.003	-0.142	-0.196	-0.030	-0.129	-0.239
	3.272	3.384	3.369	3.256	3.374	3.366
Option 5(k_n=10)	-0.106	-0.233	-0.232	-0.145	-0.209	-0.277
	3.220	3.548	3.363	3.195	3.522	3.387
Option 5(k_n=20)	-0.117	-0.242	-0.247	-0.149	-0.220	-0.242
	3.202	3.493	3.369	3.037	3.456	3.357

The top number in each cell is the skewness. The bottom number is the kurtosis.

Selection of m^* (k_n)

Figure : $\hat{v}_j^{(1)}$ for selection of m^* and k_n

(a) $d = 0.4,\ \phi = 0.3,\ m = 20$ (b) $d = 0.4,\ \phi = 0.8,\ m = 5$



Monte Carlo: $0.75 < d < 1$

$$(1-L)^{0.8} x_t = u_t \ , \quad t = 0, 1, 2, ...,$$

where $u_t \sim NID(0,1)$. This process sets $u_t = 0$ for $t \leq 0$. Then

$$x_t = \sum_{k=0}^{t-1} a_k u_{t-k}$$

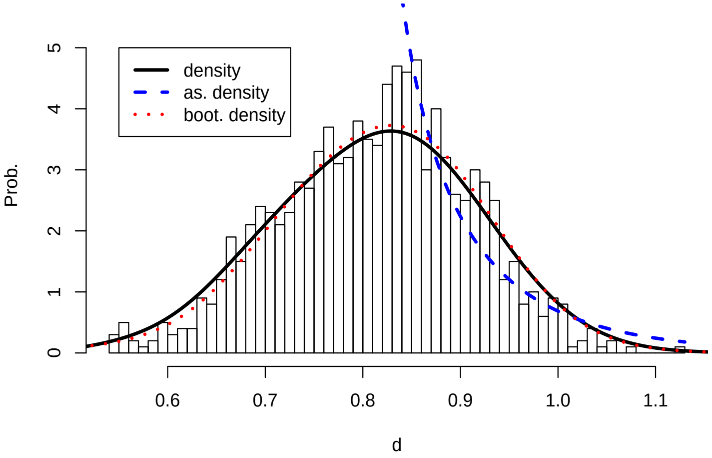
with $a_0 = 1$ and $a_k = \frac{d(d+1)\cdots(d+k-1)}{k!}$ for $k > 0$

- $n = 512$.
- $m = 20, 40, 70$.
- $m^* = 5, 10, 20$.
- $k_n = 5, 20, 40, 70$.

Nonstationary case

- Asympt. distribution poor approximation even in large samples.

Figure : Pdf LWE, ARFIMA(0,0.8,0),$n = 512$, $m = 40$



Nonstationary case

95% Confidence Intervals, g_1 and g_2

Table : CI, Skew. (g_1) and Kurt. (g_2), $ARFIMA(0, 0.8, 0)$, ($n = 512$)

	m=20	m=40	m=70	m=20	m=40	m=70
$CI^1_{0.95}$ || MC g_1, g_2	0.549	0.577	0.557	-0.233	-0.156	-0.192
	(0.496)	(0.376)	(0.300)	2.853	2.988	3.179
$CI^4_{0.95}(5)$ || g_1, g_2	0.929	0.907	0.907	-0.290	-0.226	-0.191
	(0.560)	(0.353)	(0.251)	3.404	3.248	3.164
$CI^4_{0.95}(10)$ || g_1, g_2	0.954	0.921	0.921	-0.274	-0.227	-0.190
	(0.575)	(0.361)	(0.257)	3.379	3.228	3.154
$CI^4_{0.95}(20)$ || g_1, g_2	0.974	0.952	0.924	-0.256	-0.218	-0.186
	(0.584)	(0.367)	(0.261)	3.357	3.218	3.145
$CI^5_{0.95}(5)$ || g_1, g_2	0.902	0.892	0.888	-0.204	-0.200	-0.192
	(0.523)	(0.335)	(0.240)	3.408	3.280	3.203
$CI^5_{0.95}(20)$ || g_1, g_2	0.962	0.931	0.916	-0.279	-0.229	-0.196
	(0.558)	(0.356)	(0.254)	3.414	3.245	3.157
$CI^5_{0.95}(40)$ || g_1, g_2	0.958	0.960	0.942	-0.251	-0.228	-0.194
	(0.589)	(0.361)	(0.259)	3.339	3.223	3.136
$CI^5_{0.95}(70)$ || g_1, g_2	0.968	0.951	0.954	-0.231	-0.208	-0.186
	(0.604)	(0.370)	(0.260)	3.313	3.197	3.136

Left block shows coverages (top) and lengths (bottom) of 95% CI. Right block shows skewness (top) and kurtosis (bottom). Skewness and kurtosis in first row are exact (MC) and $CI^1_{0.95}$ is obtained with the as. distribution.

Monte Carlo: Unknown as. distribution

- $ARFIMA(0, -0.7, 0)$, $(1 - L)^{-0.7} Y_t = \varepsilon_t$ for ε_t standard normal.
- $Y_t = X_t^2$ for $(1 - 0.3L)(1 - L)^{0.4} Y_t = \varepsilon_t$ and ε_t standard normal.

The sample size and bandwidth parameters are the same as those considered in the stationary case.

95% Confidence Intervals

Table : Coverages $ARFIMA(0,-0.7,0)$, sq. $ARFIMA(1,0.4,0)$ ($n=128$)

	$ARFIMA$ $(0,-0.7,0)$			squared $ARFIMA$ $(1,0.4,0)$		
	m=5	m=10	m=20	m=5	m=10	m=20
$CI^4_{0.95}(3)$	0.972	0.947	0.916	0.925	0.844	0.789
	(1.409)	(0.883)	(0.538)	(1.697)	(0.892)	(0.525)
$CI^4_{0.95}(5)$	0.995	0.959	0.929	0.952	0.892	0.812
	(1.487)	(0.917)	(0.555)	(1.751)	(0.923)	(0.542)
$CI^4_{0.95}(7)$	0.992	0.964	0.937	0.954	0.918	0.838
	(1.514)	(0.934)	(0.564)	(1.775)	(0.939)	(0.551)
$CI^5_{0.95}(2)$	0.806	0.820	0.782	0.845	0.770	0.716
	(1.163)	(0.748)	(0.476)	(1.221)	(0.739)	(0.462)
$CI^5_{0.95}(5)$	0.948	0.909	0.853	0.932	0.853	0.773
	(1.341)	(0.841)	(0.524)	(1.455)	(0.833)	(0.511)
$CI^5_{0.95}(10)$	0.988	0.955	0.901	0.972	0.910	0.823
	(1.535)	(0.886)	(0.548)	(1.722)	(0.877)	(0.532)
$CI^5_{0.95}(20)$	1.000	0.970	0.936	0.998	0.930	0.870
	(1.685)	(0.959)	(0.563)	(1.931)	(0.975)	(0.548)

The cells show coverage frequencies (top number) and lengths of CI for a 95% confidence (bottom number, in round brackets) for the noninvertible case (left block) and the nonlinear transformation (right block).

Skewness and Kurtosis

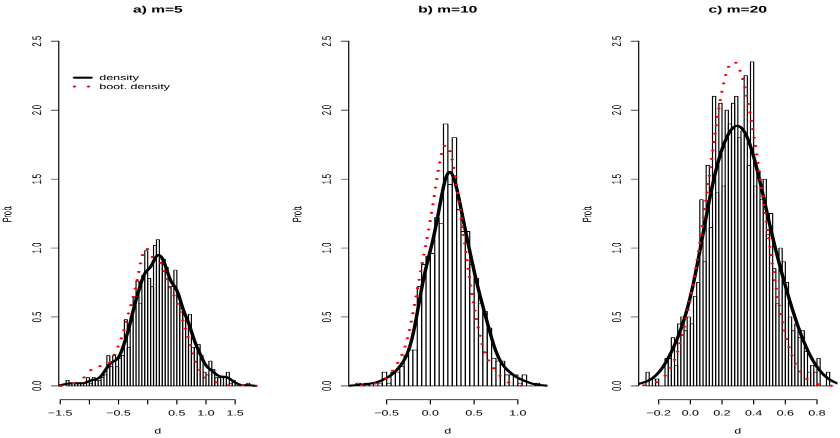
Table : Sk., Kur., $ARFIMA(0, -0.7, 0)$ and sq. $ARFIMA(1, 0.4, 0)$

	$ARFIMA\ (0, -0.7, 0)$			squared $ARFIMA\ (1, 0.4, 0)$		
	m=5	m=10	m=20	m=5	m=10	m=20
MC	0.128	-0.159	-0.252	-0.048	0.044	0.067
	2.977	3.412	3.026	3.509	3.564	2.814
Op. 4($m^* = 3$)	0.374	-0.189	-0.295	-0.262	-0.397	-0.294
	3.662	3.350	3.418	3.669	3.816	3.500
Op. 4($m^* = 5$)	0.317	-0.173	-0.291	-0.244	-0.369	-0.312
	3.422	3.271	3.371	3.638	3.755	3.469
Op. 4($m^* = 7$)	0.332	-0.166	-0.283	-0.229	-0.339	-0.310
	3.459	3.248	3.344	3.595	3.710	3.440
Op. 5($k_n = 5$)	0.306	-0.054	-0.196	0.054	-0.130	-0.187
	3.439	3.251	3.392	3.371	3.475	3.427
Op. 5($k_n = 10$)	0.380	-0.138	-0.241	-0.101	-0.264	-0.249
	3.505	3.277	3.345	3.335	3.690	3.445
Op. 5($k_n = 20$)	0.416	-0.093	-0.253	-0.088	-0.257	-0.254
	3.440	3.225	3.349	3.260	3.569	3.409

The cells show skewness (top number) and kurtosis (bottom number) for the noninvertible case (left block) and the nonlinear transformation (right block). The skewness and kurtosis in the first row are the exact ones (Monte Carlo).

Densities in squared long memory

Figure : Probability densities, squared $ARFIMA(1, 0.4, 0)$ ($n = 128$)



The bootstrap density is based on 999 bootstrap samples of the LW estimator obtained with the Studentized periodogram with $m^* = 5$.

Application to Nelson-Plosser Data

Table : LW estimator and bootstrap $CI^5_{0.95}(m^* = 5)$

	m	$\hat{d}$	$CI^4_{0.95}(5)$		$\hat{d}$	$CI^4_{0.95}(5)$	
CPI	14	0.925	0.844	1.042	0.226	-0.165	0.556
Employment	13	0.995	0.860	1.085	-0.322	-0.699	0.044
GNP deflator	11	1.083	-		0.183	-0.304	0.686
GNP per capita	11	0.964	0.711	1.131	-0.353	-0.856	0.038
Ind. production	23	0.985	0.895	1.064	-0.381	-0.579	-0.187
Bond Yield	10	1.191	-		0.132	-0.465	0.623
Money stock	20	0.982	0.923	1.004	0.328	0.051	0.605
Nominal GNP	7	0.932	0.808	1.068	0.300	-0.446	0.638
Real wages	10	1.109	-		0.068	-0.495	0.265
Real GNP	12	1.016			-0.327	-0.785	0.058
S&P500	9	0.949	0.650	1.141	-0.055	-0.928	0.204
Unemployment	7	-0.130	-0.697	0.471	-1.029	-	
Velocity	6	1.175	-		0.157	-0.747	0.632
Wages	6	1.013	-		-0.004	-0.967	1.008

The left block shows the results for raw series and the right block for differenced series.

Application to Nelson-Plosser Data: GNP per capita

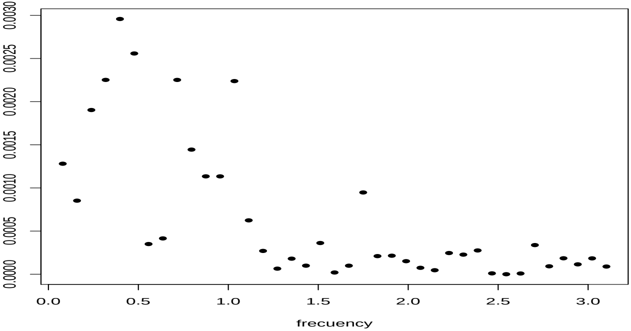
Consider for example the CI obtained with the global and local bootstrap with $m^* = 3, 5, 7$ and $k_n = 2, 5, 10$ for the first differences of the GNP per capita ($m = 11$).

Table : 95%CI for differenced GNP per capita

	$m^* = 3$	$m^* = 5$	$m^* = 7$
$CI^4(m^*)$	-0.835 -0.011	-0.856 0.038	-0.820 0.081
	$k_n = 2$	$k_n = 5$	$k_n = 10$
$CI^5(k_n)$	-0.498 -0.151	-0.536 -0.085	-0.572 0.081

Application to Nelson-Plosser Data: GNP per capita

Figure : $\hat{v}_j^{(1)}$ for selection of m^* and k_n: (differenced) GNP



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- The Studentized periodogram bootstrap seems to be more robust the choice of m^* than the locally standardized periodogram bootstrap to the selection of k_n.
- The bandwdith for LW estimation m is considered fixed. Of course m determines the exact bias and variance of the LW estimator $\Rightarrow$ can we use bootstrap for bandwidth selection?


Thank You